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A solver for the two-phase two-fluid model based on high-resolution total variation diminishing scheme

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HIGHLIGHTS

- The two-fluid model and the challenges associated with its numerical modeling are investigated.
- A high-order solver based on flux limiter schemes and the theta method was developed.
- The solver was compared to existing thermal hydraulics codes used in nuclear industry.
- The solver was shown to handle fast transients with discontinuities and phase change.

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ABSTRACT

Finite volume techniques with staggered mesh are used to develop a new numerical solver for the one-dimensional two-phase two-fluid model using a high-resolution, Total Variation Diminishing (TVD) scheme. The solver is implemented to analyze numerical benchmark problems for verification and testing its abilities to handle discontinuities and fast transients with phase change. Convergence rates are investigated by comparing numerical results to analytical solutions available in literature for the case of the faucet flow problem. The solver based on a new TVD scheme is shown to exhibit higher-order of accuracy compared to other numerical schemes. Mass errors are also examined when phase change occurs for the shock tube problem, and compared to those of the 1st-order upwind scheme implemented in the nuclear thermal-hydraulics code TRACE. The solver is shown to exhibit numerical stability when applied to problems with discontinuous solutions and results of the new solver are free of spurious oscillations.

1. Introduction

As a subject of intense interest in many engineering systems, the study of two-phase flow is of great importance in applied research because it is involved in many industrial applications. In the nuclear industry the phenomenon of two-phase flow plays a crucial role, because water in its liquid and gaseous phases is used as both a coolant and a moderator in many types of reactor cores. It also appears in other mechanical parts of the nuclear reactor, such as heat exchangers, condensers and turbines. Consequently, achieving the optimal design for both operation and safety requires a solid understanding of the fundamental aspects of two-phase flow physics and its mathematical models. Modeling of two-phase flows entails several difficulties; difficulties in the mathematical model used to govern the evolution of different properties in space and time, difficulties in physical models describing interfacial interactions, difficulties in the numerical methods used to solve the model and difficulties in building robust solvers for the algebraic equations arising from strongly non-linear discrete equations.

In the last few decades, many researchers dedicated significant effort to overcome the problems associated with modeling of two-phase flow. Because most of the legacy nuclear thermal-hydraulics codes were based on one-dimensional models, the majority of the work targeting the nuclear industry was in that area (Mousseau, 2004; Fullmer et al., 2013; Sokolowski and Koszela, 2012). Other work was performed in terms of multidimensional analysis (2D, 3D) (Balcazar et al., 2014; Bonometti and Magnaudet, 2007). Multidimensional analysis of the two-phase flow allows solving for

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different flow regimes. This entails interface capturing between the two phases using different techniques, such as the Level-Set (LS) method, the Front-Tracking (FT) method and the Volume-Of-Fluid (VOF) method. Due to its simplicity and ease of application, the VOF is the most popular technique used in CFD commercial codes like ANSYS FLUENT, STAR-CD and OpenFOAM (Ansys Inc., 2006; Lee et al., 2012).

The difficulty of physical modeling of the two-phase flow arises from the existence of moving and deforming interfaces between the two phases. Fluid properties are discontinuous at these interfaces and flow fields are complicated, there have been several mathematical models proposed for the two phase flow, one of these models is the two-fluid model used in most nuclear simulators.

Numerical difficulties depend on many factors, one of which is the desired order of accuracy. In the nuclear industry, most thermal-hydraulics codes like RELAP5 (DSR, 2010) and TRACE (DSA, 2010) employ 1st-order numerical schemes for spatial and temporal discretization. 1st-order schemes are known to possess an inherent numerical viscosity which bestows the dissipation property to the numerical solution. Such dissipation is essential to handle the solution near discontinuities. However, it causes smearing of the solution that can mask physical discontinuities and interfaces (Sweby, 1984). For higher order linear schemes, which are non-monotone, the numerical solution is always expected to pose spurious oscillations near discontinuities. This fact comes as a result of the Godunov's order barrier theorem (Godunov, 1957) which states that a linear numerical scheme for solving the advection equation, having the property of not generating new extrema (monotone scheme), can be at most 1st-order accurate.

This paper is aimed at investigating the one-dimensional two-fluid model and the challenges associated with its numerical modeling. For the numerical simulation of the problem, non-linear high-order schemes are investigated as a potential replacement for the 1st-order schemes used in nuclear thermal-hydraulic codes. Analysis for numerical properties is carried out, and a numerical solver based on this analysis is developed for the six-equation two-phase two-fluid model. We present the discrete equations of the model along with its closure relations. Temporal discretization is based on the theta method with different schemes implemented for spatial discretization including the new scheme developed in (Abu Saleem and Kozlowski, 2015). The solver is then tested and verified using numerical benchmarks and analytical solutions.

2. Mathematical model

The mathematical model for the two-phase flow is obtained by applying conservation relations to mass, momentum and energy of different phases separately. The model employed by RELAP5 and TRACE is based on the standard one-dimensional two-fluid model (DSR, 2010; DSA, 2010). This model uses a single pressure for both phases. The following two equations represent the conservation of mass for gas and liquid phases, respectively:

$$\frac{\partial(\alpha_g\rho_g)}{\partial t}+\frac{\partial(\alpha_g\rho_g u_g)}{\partial x}=\Gamma_g \tag{1}$$

$$\frac{\partial(\alpha_l\rho_l)}{\partial t}+\frac{\partial(\alpha_l\rho_l u_l)}{\partial x}=-\Gamma_g \tag{2}$$

In these equations:

- α_g and α_l are void fractions for gas and liquid phases, respectively.
- ρ_g and ρ_l, are gas and liquid densities.
- u_g and u_l are gas and liquid velocities.
- Γ_g is the interfacial mass transfer per unit volume. It is equal in magnitude and opposite in sign between the two phases. Γ_g is given by (Mousseau, 2004):

$$\Gamma_g=-\frac{H_{ig}a_{\text{int}}(T_s-T_g)+H_{il}a_{\text{int}}(T_s-T_l)}{h_g^*-h_l^*} \tag{3}$$

In this equation:

- a_{int} is the interfacial area between the two phases.
- T_g and T_l are gas and liquid temperatures, respectively.
- T_s is saturation temperature.
- H_{ig} and H_{il} are the heat transfer coefficients of gas and liquid phases with the interface.

The phasic mass transfer enthalpies of the two phases (h_g^* and h_l^*) are calculated as follows:

$$h_g^*=\begin{cases}h_{gs} & \text{if } \Gamma_g>0\\ h_g & \text{Otherwise}\end{cases}$$
$$h_l^*=\begin{cases}h_l & \text{if } \Gamma_g>0\\ h_{ls} & \text{Otherwise}\end{cases} \tag{4}$$

h_g and h_l are the phasic enthalpies of gas and liquid, respectively. h_{gs} and h_{ls} are the phasic saturation enthalpies. Conservation of momentum is also applied to each phase separately to obtain the following two equations for the gas and liquid phases, respectively:

$$\frac{\alpha_g\rho_g\partial(u_g)}{\partial t}+\alpha_g\rho_g u_g\frac{\partial(u_g)}{\partial x}+\alpha_g\frac{\partial p}{\partial x}-\alpha_g\rho_g G$$
$$=-a_{\text{int}}FI\left|u_g-u_l\right|(u_g-u_l)-\Gamma_g(u_{\text{int}}-u_g) \tag{5}$$

$$\frac{\alpha_l\rho_l\partial(u_l)}{\partial t}+\alpha_l\rho_l u_l\frac{\partial(u_l)}{\partial x}+\alpha_l\frac{\partial p}{\partial x}-\alpha_l\rho_l G$$
$$=a_{\text{int}}FI\left|u_g-u_l\right|(u_g-u_l)+\Gamma_g(u_{\text{int}}-u_l) \tag{6}$$

In the equations above G is the gravity acceleration and p is the pressure of the system. The term with the interfacial friction coefficient (FI) accounts for momentum losses due to interfacial friction. These losses are equal in magnitude and opposite in sign for the two phases. Terms including Γ in the momentum equations account for the momentum lost or gained by the new mass appearing at the interfacial velocity (u_{int}). It is assumed there is no momentum storage at the interface, therefore liquid and vapor interface velocities are equal to each other.

Conservation of energy is also applied to each phase separately. This yields the following two equations for gas and liquid phases, respectively:

$$\frac{\partial(\alpha_g\rho_g e_g)}{\partial t}+\frac{\partial(\alpha_g\rho_g e_g u_g)}{\partial x}+p\frac{\partial\alpha_g}{\partial t}+p\frac{\partial(\alpha_g u_g)}{\partial x}$$
$$=H_{ig}a_{\text{int}}(T_s-T_g)+\Gamma_g h_g^* \tag{7}$$

$$\frac{\partial(\alpha_l\rho_l e_l)}{\partial t}+\frac{\partial(\alpha_l\rho_l e_l u_l)}{\partial x}+p\frac{\partial\alpha_l}{\partial t}+p\frac{\partial(\alpha_l u_l)}{\partial x}=H_{il}a_{\text{int}}(T_s-T_l)-\Gamma_g h_l^* \tag{8}$$

In these equations, e_g and e_l are the specific energies for gas and liquid phases, respectively. Finally, conservation of volume is applied:

$$\alpha_g+\alpha_l=1 \tag{9}$$