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Robust filtering for dynamic compensation of self-powered neutron detectors

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HIGHLIGHTS

- Three dynamic compensation methods based on robust filtering theory are proposed.
- Filter design problems are converted into linear matrix inequality problems.
- Rhodium and Vanadium self-powered neutron detectors are used to validate the use of these three dynamic compensation methods.
- The numerical simulation results show that all three methods can provide a reasonable balance between response speed and noise suppression.

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ABSTRACT

Self-powered neutron detectors (SPNDs), which are widely used in nuclear reactors to obtain core neutron flux distribution, are accurate at steady state but respond slowly to changes in neutron flux. Dynamic compensation methods are required to improve the response speed of the SPNDs and make it possible to apply the SPNDs for core monitoring and surveillance. In this paper, three digital dynamic compensation methods are proposed. All the three methods are based on the convex optimization framework using linear matrix inequalities (LMIs). The simulation results show that all three methods can provide a reasonable balance between response speed and noise suppression.

1. Introduction

In nuclear reactors, the three-dimensional (3D) in-core power distribution continuously and complexly changes due to the movement of control rods, the feedback effects of reactivity, the burn-up of fuel, and etc. Thus, to ensure the safety of reactors, monitoring the in-core power distribution continuously is necessary (Peng et al., 2014).

Three types of detector have been developed to determine in-core power distribution in PWRs: ex-core neutron detector, movable detector, and fixed in-core self-powered neutron detector (SPND). Fixed in-core SPNDs can provide information of in-core power distribution with higher reliability compared with ex-core detectors. More and more nuclear reactors are using fixed in-core SPNDs in 3D power distributions on-line monitoring. The current generated by an SPND has typically two components: a prompt part and a delayed part. It is rather difficult to use the signals of these detectors for reactor control and protection due to the delayed part. Therefore, good signal processing methods are required to reconstruct the flux measured by these detectors as promptly as possible.

Previously, several filtering methods have been investigated to compensate the delayed signals of Rhodium SPNDs. Auh (1994) studied the Kalman filter method to improve the slow response of the Rhodium detectors, but there remain some difficulties in filter design such as the requirement of the prior knowledge of noise covariance. Park et al. (1999) introduced a LMI-based H_∞ filter method to relax the limitation of the prior knowledge of noise, but there remain some difficulties in determining different tuning parameters which are used to suppress the noise gain. The experimental measurements have been performed to demonstrate the applicability of the H_∞ filter method with the measured data during stepwise power reduction at HANARO research reactor in Korea (Park et al., 2008).

In this paper, three digital dynamic compensation methods based on robust filtering are investigated, and these methods have small noise gain values. The three digital dynamic compensation

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methods are the H_∞ filtering method, the H_2 filtering method and the mixed H_2/H_∞ filtering method. The H_∞ filtering method of this paper is an improved version of Park.

2. Discrete-time LMI-based robust filtering theory

State estimation has been one of the fundamental issues in the control area and there have been a lot of works following those of Kalman and Luenberger. The filter determination is carried out by defining a suitable performance index in terms of the state estimation error variance. Fundamentally, two kinds of performance indexes have been considered according to the a priori assumptions on the input noise. In the H_2 filtering approach (Geromel et al., 2000) the noise characteristics are known leading to the minimization of the H_2 norm of the transfer function from the process noise to the estimation error, while in the H_∞ filtering case (Geromel et al., 2000) the performance index to be minimized being the H_∞ norm from the process noise to the estimation error. The mixed H_2/H_∞ filtering method (Simon and El-Sherief, 1994) consists of minimizing an upper bound to the H_2 norm criterion while a prescribed noise attenuation level bounds the H_∞ norm, stipulating a kind of trade-off between the H_2 performance and the noise attenuation.

Consider the following linear time-invariant discrete time system given by

$$\begin{aligned} x_{k+1} &= Ax_k + Bw_k \\ y_k &= Cx_k + Dw_k \\ z_k &= Lx_k \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state vector, $y_k \in R^r$ is the measurements output vector, $w_k \in R^m$ is the noise signal vector (including process and measurement noise) and $z_k \in R^p$ is the signal to be estimated.

The key idea of the filtering problem is to find the estimate $\hat{z}_k$ of the signal z_k such that a performance criterion, like H_2 or H_∞ norms, is minimized in an estimation error sense. The available estimates are based on the set of the measurement output signal obtained at each time k. In this sense, the purpose is to design an asymptotically stable linear filter described by

$$\begin{aligned} \hat{x}_{k+1} &= A_f\hat{x}_k + B_f\hat{y}_k \\ \hat{z}_k &= C_f\hat{x}_k \end{aligned} \tag{2}$$

where the matrices $A_f \in R^{n\times n}$, $B_f \in R^{n\times r}$, and $C_f \in R^{p\times n}$.

Defining the state error as $e_k = x_k - \hat{x}_k$ then, the estimation error is given by $\tilde{z}_k \triangleq z_k - \hat{z}_k$, the closed-loop transfer function from the noise signal w_k to the error output $\tilde{z}_k$ is written as $H_{\tilde{z}w}(\zeta)$.

The filtering design problems to be addressed in this paper are stated as:

(1) The H_2 filtering problem: determine a stable linear filter such that the estimation error variance has an upper bound, i.e.,

$$\lim_{k\to\infty} E\{\tilde{z}_k^T\tilde{z}_k\} \le \mu \tag{3}$$

Usually, the input noise signal is supposed to have a known power spectral density matrix.

(2) The H_∞ filtering problem: determine a stable linear filter ensuring a prespecified noise attenuation level, i.e.,

$$\left\|H_{\tilde{z}w}\right\|_\infty = \sup_{w\in L_2[0,\infty)} \frac{\left\|\tilde{z}\right\|_2}{\|w\|_2} \le \gamma \tag{4}$$

Since the induced L_2 norm of the operator does not require any knowledge, expect to be bounded, the H_∞ filtering problem shows to be a powerful strategy.

(3) The mixed H_2/H_∞ filtering problem: determine a stable filter such that the estimation error variance has an upper bound, i.e., $\lim_{k\to\infty} E\{\tilde{z}_k^T\tilde{z}_k\} \le \mu$ with the bound $\left\|H_{\tilde{z}w}\right\|_\infty \le \gamma$.

2.1. The H_2 filtering

The H_2 filtering problem (Geromel et al., 2000) can be solved by solving the following linear matrix inequalities (LMIs) with a given $\mu > 0$:

$$\begin{aligned} &\mathrm{trace}[W] < \mu \\ &\begin{bmatrix} Z & Z & L^T - G^T \\ * & Y & L^T \\ * & * & W \end{bmatrix} > 0 \\ &\begin{bmatrix} Z & Z & ZA & ZA & ZB \\ * & Y & YA+FC+Q & YA+FC & YB+FD \\ * & * & Z & Z & 0 \\ * & * & * & Y & 0 \\ * & * & * & * & I \end{bmatrix} > 0 \end{aligned} \tag{5}$$

By solving these LMIs, we can obtain the matrices Q, G, F, and the symmetric positive matrices Y, Z, W, and the H_2 filter matrices can be formulated as

$$A_f = -Y^{-1}Q(I - Y^{-1}Z)^{-1}, \quad B_f = -Y^{-1}F, \quad C_f = G(I - Y^{-1}Z)^{-1} \tag{6}$$

2.2. The H_∞ filtering

The H_∞ filtering problem (Geromel et al., 2000) can be solved by solving the following linear matrix inequalities (LMIs) with a given $\gamma > 0$:

$$\begin{bmatrix} Z & Z & ZA & ZA & ZB & 0 \\ * & Y & YA+FC+Q & YA+FC & YB+FD & 0 \\ * & * & Z & Z & 0 & L^T - G^T \\ * & * & * & Y & 0 & L^T \\ * & * & * & * & I & 0 \\ * & * & * & * & * & \gamma^2 I \end{bmatrix} > 0 \tag{7}$$

By solving these LMIs, we can obtain the matrices Q, G, F, and the symmetric positive matrices Y, Z, and the H_∞ filter matrices can be formulated as

$$A_f = -Y^{-1}Q(I - Y^{-1}Z)^{-1}, \quad B_f = -Y^{-1}F, \quad C_f = G(I - Y^{-1}Z)^{-1} \tag{8}$$

2.3. The mixed H_2/H_∞ filtering

Suppose that the noise is partitioned as $[w\,v]$, the idea is to treat v as a noise signal with known spectrum (assumed without loss of generality to be white) and w as a noise signal with unknown spectrum. The linear time-invariant discrete time system can be rewritten as

$$\begin{aligned} x_{k+1} &= Ax_k + B_1w_k + B_2v_k \\ y_k &= Cx_k + D_1w_k + D_2v_k \\ z_k &= Lx_k \end{aligned} \tag{9}$$