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Solution of the multiplying binary stochastic media based on L–P equation in 1D spherical geometry

Qingjie Liu*, Hongchun Wu, Liangzhi Cao

School of Nuclear Science and Technology, Xi'an Jiaotong University, Xi'an 710049, PR China

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ABSTRACT

In order to obtain the ensemble average k_{eff} for the binary stochastic media system, a statistical transport equation with eigenvalue is derived based on the L–P equation. Combined method of statistical and deterministic is proposed to deal with this problem. In the first step, Monte Carlo approach is employed to calculate the mean chord length of the background scattering material, deterministic transport method using diamond difference and source iteration is applied to solve the eigenvalue L–P equation. Test problems of different scattering ratios, different chord-path ratios and different number of random fissile lumps and simple UO_2–H_2O problem are calculated and compared with references. Results show that the eigenvalue L–P equation can provide the mean value of k_{eff} for the given stochastic systems in most cases.

1. Introduction

Particle and radiation transport in the stochastic medium has been widely applied in radiative transfer, atmospheric sciences and considerable interests in nuclear criticality safety, nuclear waste disposal and repository, numbers of methods have been advised and developed. Statistical analyses are carried out based on transport or diffusion theory accounting for appropriate statistical approximation. The statistics of flux and multiplication factor are obtained for the random system through either analytical or numerical means (Williams, 2000, 2002, 2003). L–P model (Adams et al., 1989; Pomraning, 1989; Malvagi and Pomraning, 1992; Su and Pomraning, 1993; Sanchez, 1989, 2006, 2008a,b; Akcasu and Williams, 2004; Akcasu, 2007) is developed based on an exact stochastic equation for material fluxes and on a approximate relation. The exact equation contains interface fluxes which includes the ensemble averages over realizations that the fluxes change from one material to the other at a given spatial location. The approximate relation replaces the complicated interface fluxes by the material fluxes from the upstream material. This approximation is exact in pure-absorbing material with Markov chord length statistics and this is extended to the cases with scattering. Alternating renewal theory is chosen for non-Markov statistics. Davis and Palmer (2005) have given out the benchmark for two-group k-eigenvalue calculation in planar geometry. In their work, the problem is comprised of fuel and moderator. The size of the problem and the mean chord length of the fuel or moderator are determined by chord length sampling of a PWR assembly. Markov chord length distribution and sampling is employed to determine the random thickness of alternating fuel and moderator in the 1D slab system. Different approaches of chord length sampling are intended (e.g. for moderator, Markov or Matrix; for fuel, disk or Markov). The ensemble average flux and the PDF of the system k_{eff} is eventually obtained by performing sufficient numbers of the calculation. Nevertheless, no eigenvalue problem for multiplying media was proposed concerning straightforward solution of the LP model to estimate the ensemble average k-eigenvalue.

In this paper, an eigenvalue statistical transport equation is proposed by adding particular fission term and eigenvalue to the L–P equation. Spherical outer boundary scattering background system consists of number of randomly distributed fissile pellets is studied. Mean chord length of the background material is obtained by Monte Carlo approach under homogeneous Markov chord length distribution approximation and the ensemble average k_{eff} of the system is calculated by straightforward solution of the eigenvalue L–P equation in 1D spherical geometry.

In Section 2 the model of eigenvalue L–P equation is explained. Section 3 illustrates the generation of the random system, Section 4 provides the solution of the mean chord length. In Section 5, numerical results are given and compared with the reference. Conclusion and remarks are made in the final section.

2. Eigenvalue L–P equation

Fig. 1 shows a binary stochastic system with fissile material i randomly distributed in the non-fissile scattering background material

* Corresponding author at: School of Nuclear Science and Technology, Xi'an Jiaotong University, P.O. 2335, No. 28 Xianning West Road, Xi'an, Shaanxi 710049, PR China. Tel.: +86 29 82663285; fax: +86 29 82667802.
E-mail addresses: 89354815@qq.com, jaromir.qjliu@gmail.com (Q. Liu).

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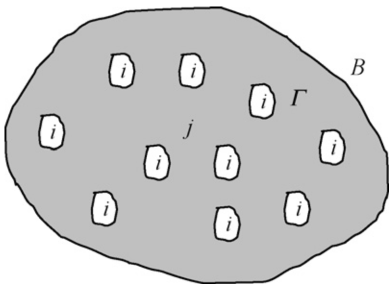

Fig. 1. Binary stochastic media system with random distributed fissile lumps.

j. B is the outer boundary of the convex volume V and Γ is the interior surface that separates two materials.

2.1. The L–P equation

$$\begin{aligned}&\Omega\cdot\nabla(\psi_i(r,\Omega))+\Sigma_{ti}\psi_i(r,\Omega)+\frac{1}{\lambda_i}(\psi_i(r,\Omega)-\psi_j(r,\Omega))\\&=\frac{1}{4\pi}\Sigma_{si}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'+Q_i(r,\Omega)\end{aligned}\tag{1}$$

Eq. (1) is the steady-state, one group, isotopic scattering L–P equation (Adams et al., 1989) for material i in binary stochastic media. It is given under the assumption of Markov mixing statistics for the two components in the system. Further, the statistics are taken as homogeneous, by which it means that all points in the system have the same statistical properties. In the L–P equation, material fluxes ($\psi_i(r,\Omega)$, $\psi_j(r,\Omega)$) are used to replace the interface fluxes ($\psi_{ij}(r,\Omega)$, $\psi_{ji}(r,\Omega)$) in the coupling term (true for pure-absorbing mixture and extended to cases with collision). The ensemble average flux is given as

$$\left\langle\psi(r,\Omega)\right\rangle=\psi_i(r,\Omega)p_i+\psi_j(r,\Omega)p_j\tag{2}$$

where for homogeneous statistics, we have

$$p_i=\frac{\lambda_i}{\lambda_i+\lambda_j},\quad p_j=\frac{\lambda_j}{\lambda_i+\lambda_j}\tag{3}$$

$\psi_i(r,\Omega)$ is the material angular flux for material i. p_i is the emerging probability that finding material i at a spatial point r in the system. The notation $\langle\cdot\rangle$ indicates ensemble averaging operation, λ_i is the mean chord length for material i and Σ_{it}, Σ_{si}, are total cross-section, scattering cross-section, for material i, respectively. $\langle\psi(r,\Omega)\rangle$ is the ensemble average angular flux.

The boundary condition of Eq. (1) is non-stochastic vacuum or reflective, written as

$$\psi_i(R,\Omega)=\begin{cases}0,\ \Omega\cdot n<0, & \text{for vacuum boudary}\\ \psi_i(R,\Omega), & \text{for reflective boudary}\end{cases}\tag{4}$$

2.2. Eigenvalue of the L–P equation

Fission term and eigenvalue are introduced to Eq. (1). We have

$$\begin{aligned}&\Omega\cdot\nabla(\psi_i(r,\Omega))+\Sigma_{ti}\psi_i(r,\Omega)+\frac{1}{\lambda_i}(\psi_i(r,\Omega)-\psi_j(r,\Omega))\\&=\frac{1}{4\pi}\Sigma_{si}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'+\frac{\nu\Sigma_{fi}}{k}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'\end{aligned}\tag{5}$$

where k is the eigenvalue of the equation, $\nu\Sigma_{fi}$ is the fission production cross-section of material i, isotropic fission is assumed. The only differences between Eq. (5) and Eq. (1) are the fission term and eigenvalue k. The detail procedure deriving Eq. (5) is not discussed here because it is similar to the derivation of Eq. (1) in sense of neutron balance.

To explain the fission term and eigenvalue, ensemble averaging is defined as follows:

$$\psi_i(r,\Omega)=\frac{\int_{K_i(r)}p(\omega)\psi_\omega(r,\Omega)d\omega}{\int_{K_i(r)}p(\omega)d\omega}=\frac{\int_{K_i(r)}p(\omega)\psi_\omega(r,\Omega)d\omega}{p_i(r)}\tag{6}$$

where $\psi_i(r,\Omega)$ is the ensemble average material flux at position r. The averaging is carried out on the subset $K_i(r)=\{\omega\in K,\omega(r)=i\}$ of realizations that have material i at position r and $\psi_\omega(r)$ is the flux for random realization ω. K represents the statistical set of all physical realizations. $p(\omega)$ denotes the density of probability for realization ω.

Performing ensemble averaging on the fission source for material i in the stochastic system, we have

$$F(r,\Omega)=\frac{\int_{K_i(r)}p(\omega)\frac{Q_w(\psi_\omega(r))}{k_\omega}d\omega}{p_i(r)}=\frac{Q_i\left(\int_{K_i(r)}p(\omega)\frac{1}{k_\omega}\psi_\omega(r)d\omega\right)}{p_i(r)}\tag{7}$$

where $Q_\omega(\bullet)$ is the fission source calculation operator and k_ω is the k_{eff} of the system for an individual realization ω. As the fission cross-section of material j is 0, fission term exists only when spatial point r is in material i, $Q_\omega(\bullet)$ is therefore replaced by $Q_i(\bullet)$. To make the material fission source and eigenvalue easy to handle, we assume that

$$\int_{K_i(r)}p(\omega)\frac{1}{k_\omega}\psi_\omega(r)d\omega\approx\frac{1}{k}\int_{K_i(r)}p(\omega)\psi_\omega(r)d\omega\tag{8}$$

Eq. (8) is finally written as

$$F(r,\Omega)=\frac{Q_i(\psi_i(r,\Omega))}{k}=\frac{1}{k}\frac{\nu\Sigma_{fi}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'}{4\pi}\tag{9}$$

Eq. (9) gives the fission term involving eigenvalue k that is invariant of realizations agreed with Eq. (5), representing ensemble average k_{eff} of the system.

2.3. Equivalence of the eigenvalue k and ensemble average k_{eff}

According to the definition of the ensemble averaging operation in Eq. (6), we can give the exact expression of the ensemble average k_{eff} of the system, viz.

$$k_{en}=\frac{\int_{K_i(r)}p(\omega)k_\omega d\omega}{\int_{K_i(r)}p(\omega)d\omega}\tag{10}$$

Inserting Eq. (8) into Eq. (7), eigenvalue k is expressed as

$$k=\frac{\int_{K_i(r)}p(\omega)Q_w(\psi_\omega(r))d\omega}{\int_{K_i(r)}p(\omega)\frac{Q_w(\psi_\omega(r))}{k_\omega}d\omega}\tag{11}$$

If we ensure that $Q_w(\psi_\omega(r))/k_\omega=1$, which can be satisfied by the normalization of the fission source term in the numerical solution of eigenvalue equation. Eq. (11) is then identical to Eq. (10) so that k is proven equivalent to the ensemble average k_{eff} of the system. Eq. (5) is readily to be solved to obtain the ensemble average k_{eff} of the system represented by eigenvalue k.

3. Construction of the stochastic media system

In the present paper, spherical outer boundary system is studied (Fig. 2). The system contains two components, the first is numbers of randomly dispersed fissile pellets and the second is the scattering background. The main steps of creating the stochastic system are: