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Spatial neutronic coupling aspects in nuclear reactors

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ABSTRACT

In this paper, an effort is made to gain insights about neutronic coupling and decoupling phenomena of nuclear reactors and its consequences on their safety and stability. The neutronic coupling and decoupling aspects are investigated using eigenvalue separation (EVS) methodology. Higher harmonic eigenvalues are calculated by the method of mode subtraction. The eigenvalue separation for a typical 1000 MWe PWR is calculated and its relations with reactor core shape and size and consequent effects on spatial stability are investigated. It is demonstrated quantitatively that it is necessary to optimize height to diameter (H/D) ratio to suppress the susceptibility to multimode oscillations and to enable ease in designing spatial control algorithm. Consequences of extreme H/D ratio are also addressed. Optimum shape of the reactor core is investigated and the evaluation of upper limit of about 1.3 for H/D ratio has been carried out for large PWR cores. Safety implications of neutronic loose coupling on departure from nucleate boiling ratio (DNBR) are also addressed.

1. Introduction

Large sized nuclear reactors are preferred to achieve economy of scale in power production. However large sized reactors show spatial instability, i.e., these reactors show deviation in power distribution under certain transients. Xenon induced oscillation is a practical problem encountered in almost all large thermal reactors. A few early studies (Henry and German, 1956; Randall et al., 1958) attracted the researchers to investigate the basic cause of this spatial instability. Neutronic decoupling among various parts of the core was found to be the fundamental cause of the spatial instability of large power reactors (Wiberg, 1965). Commonly used methodology to understand the neutronic coupling phenomena in nuclear reactors is based on comparison of characteristic sizes (Stacy, 2007). The characteristic size of the reactor core is the core size expressed in units of neutron migration length (mean travel distance of neutrons between production and absorption) of the core. Beyond certain threshold value of characteristic size, the core tends to become neutronically decoupled. The characteristic size method gives a gross idea of spatial coupling of the core and does not reveal the details of decoupling and its significance in any transient. To understand the phenomenon in detail, more sophisticated techniques have come into vogue. One of such technique is eigenvalue separation technique. In this technique, higher harmonics of the neutron diffusion equation are evaluated and their relative influence in total neutron diffusion equation forms the basis of degree of neutronic coupling. This needs evaluation and analysis of higher flux harmonics. In this regards, Kaplan (1961) devised a few techniques to analyze space-time problems using modal expansion methods. Later natural mode approximation was used (Foulke and Gyftopoulos, 1967) to simplify a few selected space dependent reactor dynamics problems. A series of experiments were conducted (Rydin, 1971) to assess the noise response of loosely coupled cores. Higher flux mode effects were analyzed to check the accuracy of single flux mode stability criteria (Rydin, 1973). An accurate, higher order relationship between reactor power tilts and eigenvalue separation was developed and verified experimentally (Bechner and Rydin, 1975). This study connected the asymmetric reactivity perturbations to static axial power tilt and eigenvalue separation. Later, the same was experimentally verified for fast reactors (Brumbach et al., 1988; Sanda et al., 1993).

In the present paper the eigenvalue separation and its connection with flux tilt and reactivity perturbation has been presented using simplified analytical expressions. The eigenvalue separation is calculated as a function of core size and shape and an optimized cylindrical core shape has been evaluated to minimize neutronic decoupling. Effect of flux tilt arising from higher harmonics, on departure from nuclear boiling ratio (DNBR) has also been reported.

In Section 2, fundamental mathematics, describing EVS as a parameter to understand neutronic coupling is explained. In Section 3, the methodology of calculating EVS and higher harmonics is reported. Section 4 presents the results of shape optimization studies for PWR core. In Section 5, the effect of flux tilt arising due to disturbances in a large PWR core on the margin to

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critical heat flux is reported. Section 6 summarizes the conclusions.

2. Eigenvalue separation (EVS) and neutronic coupling

Time dependence of neutron population in a nuclear reactor core can be studied by using neutron diffusion equation, which in operator form, can be written as:

$$\upsilon^{-1}\frac{\partial\phi}{\partial t} = (-A+B)\phi + \sum_s \chi_s^d \lambda_s C_s \tag{1}$$

where A is the destruction operator and B is the production operator. Other symbols in the equations have their usual meanings. Time dependent neutron flux, $\phi(r,t)$ can be expressed (Stacy, 2007) in terms of unperturbed neutron flux distribution, $\phi_0(r)$ and its eigenfunction, $\psi(r)$ as:

$$\phi(r,t) = \phi_0(r) + \Delta\phi(r,t) \tag{2}$$

where

$$\Delta\phi(r,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(r) \tag{3}$$

represents the deviation of neutron flux from steady state. a_n represents magnitude part of the eigenfunction and its value depends upon the magnitude of reactivity perturbations. The functions, $\psi_n(r)$ are based on higher harmonics of neutron diffusion equation. They could be any combination of trigonometric and Bessel's functions. For cylindrical shape, these harmonics could be axial or azimuthal harmonics or mixed (Fig. 1).

Neutron flux, at any instant is the sum of fundamental mode and higher harmonics with different weightage. Therefore, Eq. (2) can be written as:

$$\phi(r,t) = \text{steady state flux} + \text{contribution of higher harmonics}$$
$$(\text{axial} + \text{azimuthal}) \tag{4}$$

Ideally, contribution of these higher harmonics should be fixed under all operating conditions to maintain non-varying power distribution in course of time. This can be achieved in small cores to a great extent but it is not possible in large cores. This will be discussed in later section of this paper. Therefore in large core design, efforts are made to minimize the contribution of higher harmonics

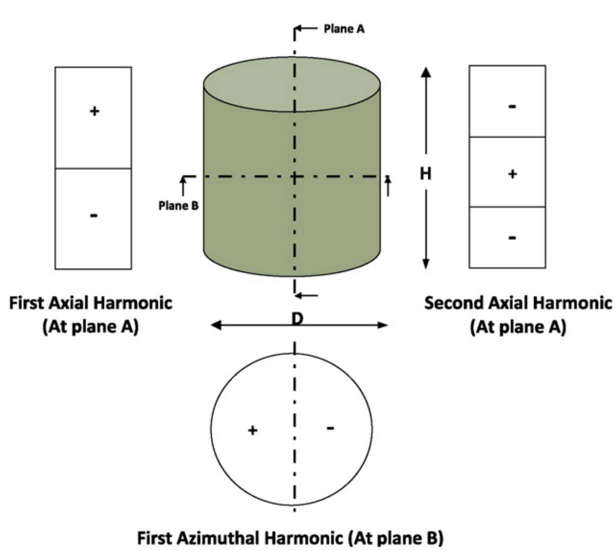

Fig. 1. Schematic of first and second harmonics.

to gain maximum inherent stability. In an operating reactor, neutron flux shape could get disturbed because of several reasons like insertion/removal of reactivity devices, xenon effects and localized perturbations due to reactivity feedbacks etc. The effect of such perturbations on power transients is different for different size of reactors. This can be demonstrated with the help of few basic equations. For ease in understanding, simplest geometry of slab reactor is considered.

2.1. Eigenvalue separation and core size

Higher harmonics eigenvalue (λ_n) of Eq. (1), can be written as:

$$\lambda_n = \frac{k_\infty}{1+M^2B_n^2} \tag{5}$$

where geometric buckling, $B_n = (n+1)\pi/a$ for a slab reactor of core thickness a. k_∞ is infinite multiplication factor, M^2 is neutron migration area, $M^2 = L^2 + \tau$, where L and τ are diffusion length and 'age to thermal neutron' respectively. Eigenvalue separation ($\in_1$), between the first mode and fundamental mode is expressed as:

$$\in_1 = \frac{1}{\lambda_1} - \frac{1}{\lambda_0} \tag{6}$$

Substituting the expressions of λ_1 and λ_0 from Eq. (5), one gets:

$$\in_1 = \frac{1+M^2B_1^2}{k\infty} - \frac{1+M^2B_0^2}{k_\infty}$$

or

$$\in_1 = \frac{M^2[(2(\pi/a))^2 - (\pi/a)^2]}{k\infty}$$

That is:

$$\in_1 \sim 3\left(\frac{M\pi}{a}^2\right) \tag{7}$$

Thus, eigenvalue separation is inversely proportional to the square of the size of the reactor core. This means, a large sized reactor core has small eigenvalue separation.

2.2. Eigenvalue separation and flux tilt

Consider a case, in which core gets subjected to asymmetric reactivity perturbation. These perturbations will disturb the neutron flux distribution. A classical way to represent such spatial perturbations is to represent them by higher mode reactivity. First mode reactivity (ρ_1) can be written as (Sanda et al., 1993):

$$\rho_1 = \frac{\langle\psi_1^*, ((\delta B/\lambda_0) - \delta A)\psi_0\rangle}{\langle\psi_1^*, B\psi_1\rangle} \tag{8}$$

where δA and δB represent changes in destruction and production operator respectively due to the reactivity perturbation, $\psi_1 \psi_1^*$ are first mode eigenfunction and adjoint eigenfunction respectively and λ_0 is fundamental mode eigenvalue. The induced flux tilt (τ) in the core due to this perturbation (ρ_1) can be written as (Sanda et al., 1993):

$$\tau = \frac{\langle\phi'(r,E)\rangle_T - \langle\phi'(r,E)\rangle_B}{\langle\phi'(r,E)\rangle} \tag{9}$$

where $\langle\,\rangle_T$ and $\langle\,\rangle_B$ and $\langle\,\rangle$ are integrals respectively over the top half, the bottom half and the total core volume. Under asymmetric perturbation, the tilt equation (Eq. (9)) can be approximated in terms of reactivity perturbation and EVS as (Sanda et al., 1993):

$$\tau = \frac{\rho_1\langle|\psi_1(r,E)|\rangle}{\in_1\langle\psi_0(r,E)\rangle} \tag{10}$$