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Free vibrations of non-uniform composite cylindrical shells

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Abstract

A semi-analytical investigation of dynamic analysis of laminated multi-layered anisotropic, open and closed cylindrical shells has been developed by taking into account the shear deformation and rotary inertia effects. The structure may be uniform or non-uniform in the circumferential direction. The method used is a combination of hybrid finite element analysis and the shearable shell theory. The shell is subdivided into cylindrical panels having 5° of freedoms at each nodal line. The set of matrices describing their relative contributions to equilibrium is determined by exact analytical integration of the equilibrium equations instead of the usually used and more arbitrarily interpolating polynomials. This theory gives zero strains for rigid-body motions in such a way that the developed displacement functions satisfy the convergence criteria of the finite element method. This theory can be used for vibration analysis of axisymmetric ($n = 0$), beam-like ($n = 1$) and shell mode ($n \geq 2$) behavior of cylindrical shells. It yields the high as well as the low eigenvalues and eigenmodes with comparable high accuracy. Reasonable agreement is found with other theories and experiments.

1. Introduction

With the widespread use in numerous industrial fields, i.e. aerospace, nuclear power, shipbuilding to name a few, design and research engineers have been compelled over the years to understand and predict more and more accurately the behaviour of shell structures under static and dynamic loading. Also, considerable attention has been focused on the design of composite shells that present the low weight and high performance structures due to their high specific strength (failure stress/weight) and specific stiffness (stiffness/weight). In the past few decades, the aircraft and aerospace industry have been pushing rapidly the advance of knowledge on composite materials. The new section (section X) of ASME boilers and pressure vessel code include use of laminate theory for the design and analysis of fibre reinforced plastic vessels to reduce the weight, save materials and enhance shipping and erection problems. Thus, the designer and researchers have been charged with the task of providing not only a shell structure with maximum reliability but also of minimum weight, which can only be justified on the basis of a careful static and/or dynamic analysis of the entire structure and a through knowledge of the material behaviour. It is known that laminated composite shells exhibit large thickness effects than the corresponding structures made of homogenous isotropic materials. The application of classical theories to layered composite shells could lead to large errors in deflection, stress and frequencies.

The study of free vibration behaviour of isotropic cylindrical shells has been carried out by many investigators and is well documented by (Leissa, 1973). The most of these works were developed originally for thin elastic shells, in both linear and non-linear cases. They are based on the classical shell theory (CST), which could lead to gross errors in the prediction of transverse deflections; natural frequencies and buckling load of composite or moderately thick shells due to the neglect of transverse shear deformations.

These errors are even higher for plates and shells made of advanced composite materials like graphite-epoxy and boron-epoxy, where the ratio of elastic modulus to shear modulus is very large (e.g. of the order 25–40 instead of 2.6 for isotropic materials). The shear deformation effect plays a much more important role in reducing the effective stiffness of anisotropic laminated composite plates and shells.

A number of theories for layered anisotropic shells exist in the literature. Many of these theories were developed for thin shells

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Nomenclature

A_i, B_i, C_i, ... and J_i ($i = 1, 2, \ldots, 10$) defined by matrix components of J.
f_i coefficients of the characteristic equation
h_i ($i = 1, 2$) scale factors
L length of the shell
L_i differential operator of equations of motion
m axial mode number
M_{xx}, $M_{x\theta}$, $M_{\theta\theta}$, $M_{\theta x}$ the moment resultants
n circumferential wave number
N_{xx}, $N_{x\theta}$, $N_{\theta\theta}$, $N_{\theta x}$ the in-plane force resultants
P_{ij} terms of elasticity matrix
Q_{xx}, $Q_{\theta\theta}$ the transverse force resultants
R mean radius of shell
t thickness of the shell
u_x, u_θ, w the axial, circumferential and radial displacements, respectively,
x axial coordinate
β_x, β_θ the rotation of tangents to the reference surface
η_i complex roots of characteristic equation
ε_i deformation vector components
σ_i stress vector components
θ circumferential coordinate
ϕ_T angle for the whole open shell
ω natural frequency
Ω non-dimensional frequency
ρ density of the shell material

List of matrices
$\{C\}$ vector of arbitrary constants
$[k]$ local stiffness matrix
$[K]$ global stiffness matrix
$[m]$ local mass matrix
$[M]$ global mass matrix
$[N]$ shape function matrix
$[T]$ transformation matrix
$\{\delta_i\}$ degrees of freedom at node i

and are based on the Kirchhoff–Love hypothesis. The first analysis that incorporated the bending-stretching coupling (due to asymmetric lamination in composites) was by (Ambartsumyan, 1964). In this analysis, he assumed that the individual orthotropic layers were oriented such that the principal axes of material symmetry coincided with the principal co-ordinates of the shell reference surface. He has written extensively on the matter, basing his work on Love's theory with some discussion of transverse stresses.

A survey of the analysis of multi-layered composite shells using Reissner's mixed variational principle was carried out by Grigolyuk and Kulikov (1988). They maintain that laminated anisotropy assumes perfect bonding between layers, and that the interply adhesive has infinitesimal thickness but infinite stiffness. This approach leads to classical laminated plate theory (CLPT) and the reference by (Jones, 1975) to CLPT is based on the Kirchhoff–Love assumptions. Vinson (1993) outlines the behaviour of shell structures made of isotropic and composites materials. The shell theory of Reissner and Naghdi is used throughout this monograph. Noor and Peters (1987) presented an analysis of the free vibration of laminated anisotropic shells of revolution and the sensitivity of their response to anisotropic material coefficients. Noor and Peters' analytical formulation is based on a form of the Sanders–Budiansky shell theory, including the effects of both transverse shear deformation and the laminated anisotropic material response.

The accuracy of a solution obtained by the finite element displacement formulation depends on whether the assumed functions accurately model the deformation modes of the given structure. To satisfy this criterion, this paper attempts to develop a hybrid element in such a way that the stiffness and mass matrices are analytically determined. The method is a combination of the conventional finite element approach and accuracy of the exact displacement functions that are determined based on shearable shell theory. The results show that the vibration characteristics could be obtained to very good accuracy with only 10 elements for different range of geometrical and physical parameters for isotropic and anisotropic materials. These vibration characteristics can be obtained for different cases of circumferential mode numbers, axisymmetric ($n = 0$), beam-like ($n = 1$) and shell modes ($n \geq 2$). It should be noted that the lowest frequencies are not generally associated with two modes ($n = 0$, $n = 1$) because of the deformations in these modes involve more strain energy than do deformations in modes where ($n \geq 2$). This theory can be also applied for analysis of circumferentially non-uniform cylindrical shells.

2. Basic theory and method

The combination of first-order transverse shear deformation theory (SDT) of shells and hybrid finite element method has been used to develop the dynamic equations of motion. This theory is based on the following assumptions:

(a) Linear elastic behaviour of laminated anisotropic materials.
(b) The shell is thin; therefore, it is assumed that the thickness-direction normal stress is negligible compared with stress tangential to the shell surface.
(c) Use the Green's exact strain–displacement relations that are expressed in arbitrary orthogonal curvilinear co-ordinates.
(d) The first order transverse shear deformation theory of the shells is adopted.

2.1. Kinematics

The general strain–displacement relations are expressed in arbitrary orthogonal curvilinear co-ordinates to define the strain–displacement relations. The physical strain vector in the orthogonal curvilinear coordinate system is defined as follows:

$$\varepsilon_{ij} = \frac{\gamma_{ij}}{h_i h_j} \quad (1)$$