Pacific Science Review A: Natural Science and Engineering 17 (2015) 87–96

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Pacific Science Review A: Natural Science and Engineering

journal homepage: www.journals.elsevier.com/pacific-science-review-a-natural-science-and-engineering/

Different types of products on intuitionistic fuzzy graphs

Sankar Sahoo*, Madhumangal Pal

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721 102, India

ARTICLE INFO

Article history:
Available online 22 March 2016

Keywords:
Intuitionistic fuzzy graphs
Direct product
Semi-strong product
Strong product
Product intuitionistic fuzzy graphs

ABSTRACT

In this paper, we define three operations on intuitionistic fuzzy graphs, viz. direct product, semi-strong product and strong product. In addition, we investigated many interesting results regarding the operations. Moreover, it is demonstrated that any of the products of strong intuitionistic fuzzy graphs are strong intuitionistic fuzzy graphs. Finally, we defined product intuitionistic fuzzy graphs and investigated many interesting results.

1. Introduction

Graph theory has applications in many areas of computer science, including data mining, image segmentation, clustering, image capturing, and networking. An intuitionistic fuzzy set is a generalisation of the notion of a fuzzy set. Intuitionistic fuzzy models provide more precision, flexibility and compatibility to the system compared to the fuzzy models.

In 1983, Atanassov [6,7] introduced the concept of intuitionistic fuzzy set as a generalisation of fuzzy sets. Atanassov added new components that determine the degree of non-membership in the definition of fuzzy set. The fuzzy sets give the degree of membership, while intuitionistic fuzzy sets give both the degree of membership and the degree of non-membership, which are more or less independent from each other; the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields, including computer science, engineering, mathematics, medicine, chemistry, and economics.

In 1975, Rosenfeld [19] discussed the concept of the fuzzy graph, the basic idea of which was introduced by Kauffman [9] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld; he developed the structure of fuzzy graphs and obtained analogues of several graphs theoretical concepts. Atanassov introduced the concept of the intuitionistic fuzzy relation. Different types of intuitionistic fuzzy graphs and their applications can be found in several papers. In addition, Sahoo and Pal [20] discussed the concept of the intuitionistic fuzzy competition graph. In this study, the direct product, semi-strong product and strong product of two intuitionistic fuzzy graphs are defined, and many interesting results involving these operations are investigated. Moreover, we defined product intuitionistic fuzzy graphs and investigated their many interesting properties.

2. Preliminaries

2.1. Graphs

A graph is an ordered pair $G=(V,E)$, where V is the set of all vertices of G (which is non empty), and E is the set of all edges of G. Two vertices x,y in a graph G are said to be adjacent in G if (x,y) is an edge of G. A simple graph is a graph without loops and multiple edges. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge. The complete graph on n vertices has $n(n-1)/2$ edges.

An isomorphism of graphs G_1 and G_2 is a bijection f between the sets of vertices of G_1 and G_2, such that any two vertices v_1 and v_2 of G_1 are adjacent in Ref. G_1 if and only if $f(v_1)$ and $f(v_2)$ are adjacent in Ref. G_2. Isomorphic graphs are denoted by Ref. $G_1 \cong G_2$.

Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two simple graphs. Next, the direct product of G_1 and G_2 is a graph $G_1 \sqcap G_2=(V,E)$ with $V=V_1 \times V_2$ and $E=\{((u_1,v_1),(u_2,v_2))|(u_1,u_2)\in E_1,(v_1,v_2)\in E_2\}$. The union of graphs G_1 and G_2 is defined as $G_1 \cup G_2=(V_1 \cup V_2, E_1 \cup E_2)$ and join is the simple graph $G_1+G_2=(V_1 \cup V_2, E_1 \cup E_2 \cup E')$, where E' is the set of all edges joining the vertices of V_1 and V_2, also assume that. $V_1 \cap V_2=\phi$.

2.2. Intuitionistic fuzzy graphs

An intuitionistic fuzzy set A on the set X is characterised by a mapping $m:X\rightarrow[0,1]$ (which is known as a membership function) and $n:X\rightarrow[0,1]$ (which is called a non-membership function). An

* Corresponding author.
E-mail addresses: ssahoovu@gmail.com (S. Sahoo), mmpalvu@gmail.com (M. Pal).
Peer review under responsibility of Far Eastern Federal University, Kangnam University, Dalian University of Technology, Kokushikan University

http://dx.doi.org/10.1016/j.psra.2015.12.007

intuitionistic fuzzy set is denoted by Ref. $A = (X, m_A, n_A)$. The membership function of the intersection of two intuitionistic fuzzy sets $A = (X, m_A, n_A)$ and $B = (X, m_B, n_B)$ is defined as $m_{A\cap B} = \min\{m_A, m_B\}$ and the non-membership function $n_{A\cap B} = \max\{n_A, n_B\}$. We write $A = (X, m_A, n_A) \subseteq B = (X, m_B, n_B)$ (intuitionistic fuzzy subset) if $m_A(x) \le m_B(x)$ and $n_A(x) \ge n_A(x)$ for all $x \in X$.

Definition 1. *The support of an intuitionistic fuzzy set $A = (X, m_A, n_A)$ is defined as $Supp(A) = \{x \in X : m_A(x) \neq 0$ and $n_A(x) \neq 1\}$. In addition, the support length is $SL(A) = |Supp(A)|$.*

Definition 2. *The core of an intuitionistic fuzzy set $A = (X_A, m_A, n_A)$ is defined as $Core(A) = \{x \in X : m_A(x) = 1$ and $n_A(x) = 0\}$. In addition, the core length is $CL(A) = |Core(A)|$.*

Now, we defined the height of an intuitionistic fuzzy set below:

Definition 3. *The height of an intuitionistic fuzzy set $A = (X, m_A, n_A)$ is defined as $h(A) = \left(\sup_{x\in X} m_A(x), \inf_{x\in X} n_A(x)\right) = (h_m(A), h_n(A))$.*

Here, an intuitionistic fuzzy graph is defined below:

Definition 4. *An intuitionistic fuzzy graph is of the form $G = (V, E, \sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2)$, $\mu = (\mu_1, \mu_2)$ and*

(i) *$V = \{v_0, v_1, \ldots, v_n\}$ such that $\sigma_1 : V \to [0,1]$ and $\sigma_2 : V \to [0,1]$ denote the degree of membership and non-membership of the vertex $v_i \in V$, respectively, and $0 \le \sigma_1(v_i) + \sigma_2(v_i) \le 1$ for every $v_i \in V$ $(i = 1, 2, \ldots, n)$.*

(ii) *$\mu_1 : V \times V \to [0,1]$ and $\mu_2 : V \times V \to [0,1]$, where $\mu_1(v_i, v_j)$ and $\mu_2(v_i, v_j)$ denote the degree of membership and non-membership value of the edge (v_i, v_j), respectively, such that $\mu_1(v_i, v_j) \le \min\{\sigma_1(v_i), \sigma_1(v_j)\}$ and $\mu_2(v_i, v_j) \le \max\{\sigma_2(v_i), \sigma_2(v_j)\}$, $0 \le \mu_1(v_i, v_j) + \mu_2(v_i, v_j) \le 1$ for every edge (v_i, v_j).*

Now, we give an example of the intuitionistic fuzzy graph:

***Example* 1** *Let $G = (V, \sigma, \mu)$ be an intuitionistic fuzzy graph, where $\sigma(v) = (\sigma_1(v), \sigma_2(v))$, $\mu(u,v) = (\mu_1(u,v), \mu_2(u,v))$. Let the vertex set be $V = \{v_1, v_2, v_3, v_4\}$ and $\sigma(v_1) = (0.3, 0.6)$, $\sigma(v_2) = (0.8, 0.2)$, $\sigma(v_3) = (0.2, 0.8)$, $\sigma(v_4) = (0.5, 0.4)$; $\mu(v_1, v_2) = (0.25, 0.45)$, $\mu(v_2, v_3) = (0.18, 0.75)$, $\mu(v_3, v_4) = (0.15, 0.52)$, $\mu(v_4, v_1) = (0.3, 0.25)$, $\mu(v_1, v_3) = (0.2, 0.8)$, $\mu(v_2, v_4) = (0.4, 0.35)$. The corresponding intuitionistic fuzzy graph is shown in Fig. 1.*

Definition 5. *[1] An intuitionistic fuzzy graph $G = (V, E, \sigma, \mu)$ is said to be complete if $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j)$ for all $v_i, v_j \in V$.*

Definition 6. *[11] The complement of an intuitionistic fuzzy graph $G = (V, E, \sigma, \mu)$ is an intuitionistic fuzzy graph $\overline{G} = (V, E, \overline{\sigma}, \overline{\mu})$ where $\sigma = \overline{\sigma}$, $\overline{\mu}_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j) - \mu_1(v_i, v_j)$ and $\overline{\mu}_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j) - \mu_2(v_i, v_j)$ for all $v_i, v_j \in V$.*

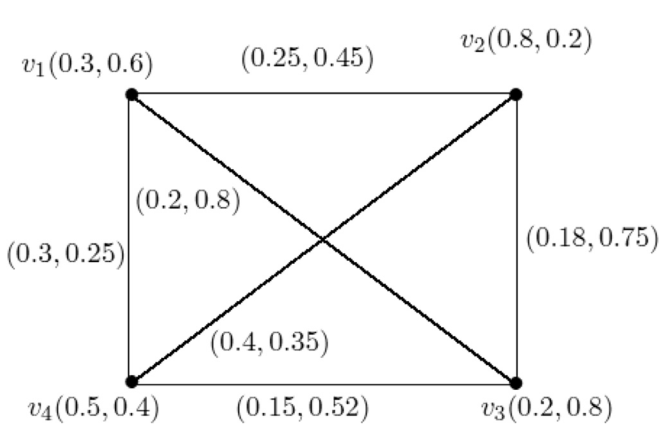

Fig. 1. An intuitionistic fuzzy graph.

2.3. Review of the literature

After Rosenfeld [19], the fuzzy graph theory increases with its various types of branches, such as fuzzy tolerance graph [25], fuzzy threshold graph [24], bipolar fuzzy graphs [17,18,30], highly irregular interval valued fuzzy graphs [12,14], isometry on interval-valued fuzzy graphs [16], balanced interval-valued fuzzy graphs [10,15], fuzzy k-competition graphs and p-competition fuzzy graphs [28], fuzzy planar graphs [23,31], bipolar fuzzy hypergraphs [26,27], and $m-$ step fuzzy competition graphs [22]. The fuzzy graph theory is used in telecommunication system [29]. A new concept of fuzzy colouring of fuzzy graph is given in Ref. [32]. Ramaswamy and Poornima [13] discussed product fuzzy graphs.

Akram and Davvaz [1] defined strong intuitionistic fuzzy graphs. They also discussed intuitionistic fuzzy hypergraphs with applications [3]. A novel application of intuitionistic fuzzy digraphs is given by Akram et al. [2]. In addition, Akram and Al-Shehrie [4] defined intuitionistic fuzzy cycles, intuitionistic fuzzy trees and intuitionistic fuzzy planar graphs [5]. Balanced intuitionistic fuzzy graphs are discussed by Karunambigai et al. [8]. Moreover, Parvathi, Karunambigai [11] defined intuitionistic fuzzy graphs. Sahoo and Pal [20] discussed the concept of intuitionistic fuzzy competition graph. They also discussed intuitionistic fuzzy tolerance graphs [21].

2.4. Our contribution

In Section 3, we define the direct product of two intuitionistic fuzzy graphs and demonstrate that the direct product of two strong intuitionistic fuzzy graphs is strong; consequently, if the direct product is strong, then any one of two intuitionistic fuzzy graphs is strong. Next, we define semi-strong, strong product of two intuitionistic fuzzy graphs and many interesting properties. Finally, in Section 4 we discuss the product intuitionistic fuzzy graph and investigated many interesting results.

3. Products on intuitionistic fuzzy graphs

In this section, we define three operations on the intuitionistic fuzzy graphs, viz. direct product, semi-strong product and strong product.

3.1. Direct product of two intuitionistic fuzzy graphs

First, the direct product of two intuitionistic fuzzy graphs is defined.

Definition 7. *The direct product of two intuitionistic fuzzy graphs $G' = (V', E', \sigma', \mu')$ and $G'' = (V'', E'', \sigma'', \mu'')$ such that $V' \cap V'' = \phi$, is defined to be the intuitionistic fuzzy graph $G' \sqcap G'' = (V, E, \sigma' \sqcap \sigma'', \mu' \sqcap \mu'')$ where $V = V' \times V''$, $E = \{((u_1, v_1), (u_2, v_2)) | (u_1, u_2) \in E', (v_1, v_2) \in E''\}$. The membership and non-membership values of the vertex (u,v) in $G' \sqcap G''$ are given by*

$$(\sigma_1' \sqcap \sigma_1'')(u,v) = \sigma_1'(u) \wedge \sigma_1''(v)$$

and

$$(\sigma_2' \sqcap \sigma_2'')(u,v) = \sigma_2'(u) \vee \sigma_2''(v).$$