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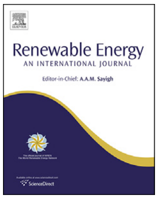

A new analytical model for wind-turbine wakes

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ABSTRACT

A new analytical wake model is proposed and validated to predict the wind velocity distribution downwind of a wind turbine. The model is derived by applying conservation of mass and momentum and assuming a Gaussian distribution for the velocity deficit in the wake. This simple model only requires one parameter to determine the velocity distribution in the wake. The results are compared to high-resolution wind-tunnel measurements and large-eddy simulation (LES) data of miniature wind-turbine wakes, as well as LES data of real-scale wind-turbine wakes. In general, it is found that the velocity deficit in the wake predicted by the proposed analytical model is in good agreement with the experimental and LES data. The results also show that the new model predicts the power extracted by downwind wind turbines more accurately than other common analytical models, some of which are based on less accurate assumptions like considering a top-hat shape for the velocity deficit.

1. Introduction

Due to the fast growth in the number and size of installed wind farms around the world, wind-turbine wakes have become important topics of study. As many wind turbines in wind farms have to operate in the wakes of upwind turbines, they are exposed to incoming wind velocities that are smaller than those under unperturbed (unwaked) conditions. As a result, turbine wakes are responsible for important power losses in wind farms [1–3]. Extensive analytical, numerical and experimental efforts have been carried out to better understand and predict turbine wake flows. Although numerical and experimental techniques have become increasingly sophisticated and accurate in recent years, simple analytical models are still useful tools to predict wind-turbine wake flows and their effect on power production. They are widely used due to their simplicity and low computational cost [4]. Various analytical investigations have been conducted on wind-turbine wakes (e.g., [5–7]). One of the pioneering analytical wake models is the one proposed by Jensen [8], which assumes a top-hat shape for the velocity deficit in the wake (see Fig. 1a) and states:

$$\frac{\Delta U}{U_\infty} = \left(1-\sqrt{1-C_T}\right)\Big/\left(1+\frac{2k_{\mathrm{wake}}x}{d_0}\right)^2, \tag{1}$$

where C_T is the thrust coefficient of the turbine, k_{wake} the rate of wake expansion, d_0 the diameter of the wind turbine and x the downwind distance. $\Delta U/U_\infty$ is the normalized velocity deficit, which is defined as:

$$\frac{\Delta U}{U_\infty} = \frac{U_\infty - U_w}{U_\infty}, \tag{2}$$

where U_∞ is the incoming wind velocity and U_w the wake velocity in the streamwise direction. Jensen [8] considered a constant value for the rate of wake expansion ($k_{\mathrm{wake}} = 0.1$). However, the suggested values for k_{wake} in the literature are 0.075 [9] for on-shore cases and 0.04 [10,11] or 0.05 [9,12] for off-shore ones. Katić et al. [5] also used the top-hat model proposed by Jensen [8]. They claimed that the top-hat model gives an estimate of the energy content rather than describing the velocity field accurately, and hence they adopted a top-hat shape for the velocity deficit in the wake because of its simplicity and low computational cost. Nevertheless, note that the energy available in the wind varies as the cube of the wind speed [13] and, therefore, an improper evaluation of velocity field in a wind farm can lead to large errors in the prediction of the energy output. This will be discussed in detail in Section 3.

Eq. (1) has been extensively used in the literature (e.g., Marmidis et al. [14]) and commercial softwares such as WAsP [9], WindPRO [15], WindSim [16], WindFarmer [17] and OpenWind [18]. However, there are two important limitations of this simple model that should be pointed out: (a) The assumption of the top-hat distribution of the velocity deficit is not realistic [19,20]. (b) Even though

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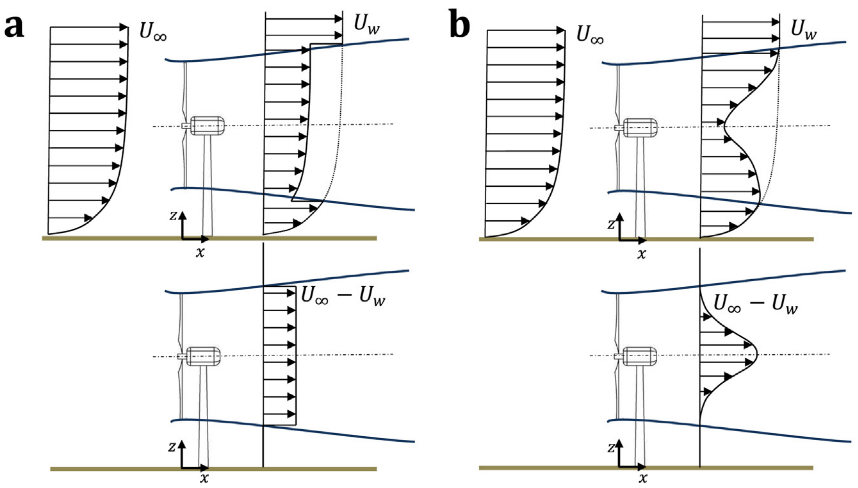

Fig. 1. Schematic of the vertical profiles of the mean velocity (top) and velocity deficit (bottom) downwind of a wind turbine obtained by assuming: (a) a top-hat and (b) a Gaussian distribution for the velocity deficit in the wake.

Jensen [8] and Katić et al. [5] claimed using momentum conservation to derive Eq. (1), it will be shown in the following that in reality they only used mass conservation to derive their model.

Jensen [8] considered a control volume immediately downwind of the turbine. Fig. 2a shows a schematic of this control volume with the left cross-sectional area (side 1) equal to the area swept by the wind-turbine blades, A_0, and the right area (side 3) equal to the cross-sectional area of the wake, A_w. The incoming flow also enters into the control volume through the lateral surface (side 2) with the velocity of U_∞. According to mass conservation:

$$\dot{m}_2 = \rho U_\mathrm{w} A_\mathrm{w} - \rho U_\mathrm{a} A_0, \tag{3}$$

where $\dot{m}_2$ is the mass flow rate through the lateral surface, ρ the density of the air and U_a the wind velocity just behind the wind turbine (see Fig. 2a). Note that if $\dot{m}_2$ is replaced with $\rho U_\infty (A_\mathrm{w} - A_0)$ in the mass conservation equation (Eq. (3)), without considering momentum conservation, the basic equation that Jensen [8] used to establish his model will be obtained. It implies, therefore, that this model can be derived by considering mass conservation alone without any consideration of the balance of momentum.

Later, Frandsen et al. [21] applied mass and momentum conservation to a control volume around the turbine (Fig. 2b) and proposed the following expression for the velocity deficit in the wake:

$$\frac{\Delta U}{U_\infty} = \frac{1}{2}\left(1 - \sqrt{1 - 2\frac{A_0}{A_\mathrm{w}} C_\mathrm{T}}\right), \tag{4}$$

where $A_\mathrm{w}(x = 0) = A_\mathrm{a}$, and A_a is the cross-sectional area of the wake just after the initial wake expansion. In other words, they assumed that the distance downwind of a rotor that the flow requires to reach the pressure of the free flow is negligible, so they considered A_a as the wake cross-sectional area at $x = 0$. It is, however, difficult to identify exactly this distance in reality. Crespo et al. [4] stated that the length of this region is in the order of one rotor diameter. Even though this assumption is crude, it ensures a solution for all C_T values between 0 and 1 [21]. According to the actuator disk concept [13], A_a is given by:

$$A_\mathrm{a} = \beta A_0, \tag{5}$$

where β is a function of C_T and can be expressed as:

$$\beta = \frac{1}{2}\frac{1 + \sqrt{1 - C_\mathrm{T}}}{\sqrt{1 - C_\mathrm{T}}}. \tag{6}$$

They also used an asymptotic solution for an infinite row of two-dimensional obstacles to write the wake diameter, d_w, as:

$$d_\mathrm{w} = \left(\beta + \alpha x / d_0\right)^{1/2} d_0, \tag{7}$$

where the expansion factor α is of order 10 k_wake [21]. While Frandsen et al. [21] employed the mass and momentum equations, their model still assumed a top-hat shape for the velocity deficit in the wake.

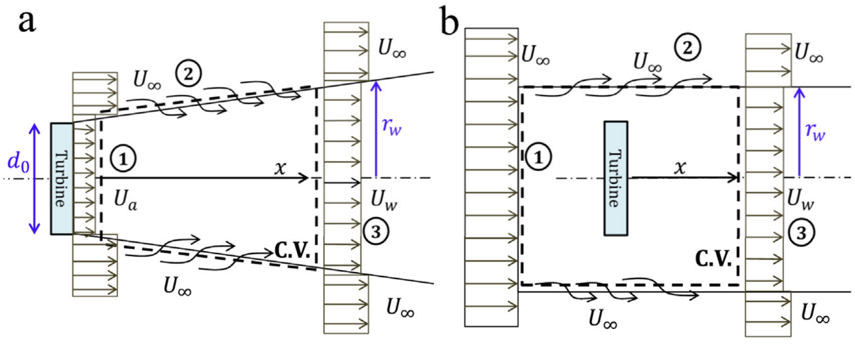

Fig. 2. Schematic of the two control volumes: (a) downwind of the wind turbine, and (b) around the wind turbine.