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On the generation of vorticity by force fields in rotor- and actuator flows

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ABSTRACT

In most rotor design methods, the blade load is found by a blade element analysis in an iterative procedure with flow solvers like actuator disc and -line analyses as well as momentum balances. For the flow solvers the force field is the input. In most other aerodynamic analyses the force field is the output result instead of input. This is done by applying boundary conditions at the lifting surface with which the flow is solved and the pressure at the surface, so the load, is determined (only inviscid flows are considered here). Both approaches are consistent, but appear to differ with respect to the generation of vorticity. In the lifting surface approach, usually Helmoltz's laws are used to show that bound and free vorticity is conserved instead of being generated, while in the force field approach vorticity is generated instead of conserved. It is shown that both methods are consistent since sometimes Helmholtz's laws are incorrectly referred to. These laws have been derived in absence of non-conservative forces, while the surface pressure distribution is shown to be such a force field. Besides this, the question is discussed how a force field creates vorticity in an inviscid flow, since some papers consider viscosity to be necessary to generate vorticity. A literature study contradicts this, showing that in inviscid flows vorticity is generated by tangential pressure gradients or, equivalently, a non-uniform force field. This makes the Euler equation including the force field term well suited to express the generation of vorticity in characteristics of the force field. A comparison of the convection of vorticity in the wake of a disc, rotor blade and wing shows several differences. The azimuthal vorticity in the disc wake does not depend on vorticity conservation laws, in contrast to the axial and radial components. For a rotor and wing all components are governed by vorticity conservation.

1. Introduction

Rotor aerodynamics is one of the few areas in aerodynamics where forces fields are used as input in flow calculations. In most other aerodynamic analyses the force field is the output instead of input. One of the reasons why force fields as input are not used any more is that usually they are not known in advance. Furthermore the kinematical method for which Lanchester, Prandtl and Joukowsky laid the basis, has been shown to be very powerful. However, particularly in rotor aerodynamics the use of force fields has some advantages. The force field approach allows for much easier physical interpretation of flow problems, since the thrust, being the integrated load, is the main parameter defining flow states. This holds for the classical actuator disc theory, the Blade Element Momentum (BEM) methods and also for actuator line analyses. Herein the blade is replaced by a load carrying line at the quarter chord position in order to have a much lower computation time compared to full Computational Fluid Dynamics (CFD) solutions. The load is determined either by the definition of the problem (in actuator disc analyses: based on physical arguments a load distribution is assumed, e.g. Sørensen, Shen, Munduate [20]) or by iteration with other methods (in actuator line methods and in methods based on momentum balances: for a given flow field the load is taken from a Blade Element calculation, e.g. Shen, Zhu, Sørensen [19]).

The kinematical approach (no force field, boundary conditions at the lifting surface) and the force field approach (external force fields) are compatible, as shown by Prandtl [16] for a wing and by van Kuik [9] for a rotor blade. However, a comparison of both methods with respect to the generation and convection of vorticity is not yet available while an important difference is observed, at first sight. A force field $\boldsymbol{f}$ is known to generate vorticity when $\nabla \times \boldsymbol{f} \neq 0$ so when it is non-conservative. For a uniform load distribution this is the case at the edge of an actuator disc or at the root and tip of an actuator line: at these positions vorticity is produced and trailed into the flow. In the kinematical method the blade is the carrier of bound vorticity, which continues as trailing vorticity. In other words: vorticity is conserved instead of generated which is

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sometimes explained by the Helmholtz laws for vorticity conservation. The consistency of both methods with respect to the generation of vorticity is the first topic of this paper. Moreover the question is addressed how a force field can produce vorticity in an inviscid flow, since authors like Betz [2] mention viscosity as the main source of vorticity.

A second topic is a comparison of the vorticity convection in the wake of a disc, rotor and wing with respect to its conservation, and of the use of linearised models.

The next section treats the generation of vorticity by force fields, after which the convection of the vorticity is discussed in Section 3, followed by a concluding section.

2. Generation of vorticity

2.1. The equations of motion

The flow is assumed to be incompressible, homogeneous, inviscid and isentropic. Furthermore only steady flows are discussed here so the Euler equation:

$$(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \frac{1}{\rho}(\boldsymbol{f} - \nabla p) \tag{1}$$

is valid, with $\boldsymbol{v}$ being the velocity vector, ρ being the flow density and p the pressure, as well as the continuity equation

$$\nabla\cdot\boldsymbol{v} = 0. \tag{2}$$

The vector identity $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \nabla(1/2\boldsymbol{v}\cdot\boldsymbol{v}) - \boldsymbol{v}\times\omega$ converts (1) to:

$$\boldsymbol{f} = \nabla H - \rho\boldsymbol{v}\times\boldsymbol{\omega} \tag{3}$$

where H is the Bernoulli constant $p + \rho/2\boldsymbol{v}\cdot\boldsymbol{v}$ and ω the vorticity.

The use of the force field $\boldsymbol{f}$ is discussed in old textbooks and papers, like von Kármán & Burgers [6]. Most modern textbooks pay some attention to the force term but at some moment assume that $\boldsymbol{f}$ is conservative, like the gravity force field. A conservative force field satisfies $\nabla\times\boldsymbol{f} = 0$ or equivalently:

$$\boldsymbol{f} = -\nabla\mathcal{F}, \tag{4}$$

with $\mathcal{F}$ being the potential of $\boldsymbol{f}$. For this reason a conservative $\boldsymbol{f}$ is sometimes mentioned a potential force field. Most textbooks assume $\nabla\mathcal{F}$ to be included in the pressure gradient ∇p, by which the force field term disappears from the equation of motion.

Here the force field term is retained explicitly. The force density $\boldsymbol{f}$ is confined to a limited volume V: force fields acting throughout space such as gravity fields are excluded. The relation between the force density $\boldsymbol{f}$ [Nm^{-3}], and the surface load $\boldsymbol{F}$ [Nm^{-2}], is defined by integration of $\boldsymbol{f}$ across the thickness ε:

$$\int_\varepsilon \boldsymbol{f}\mathrm{d}x = \boldsymbol{F}. \tag{5}$$

In general, the force field can have both components, non-conservative as well as a conservative. A non-conservative force field $\boldsymbol{f}$ is able to generate vorticity, as shown by the curl of (3), see Saffman [18] p. 10–11:

$$(\boldsymbol{v}\cdot\nabla)\omega = \frac{1}{\rho}\nabla\times\boldsymbol{f} + (\omega\cdot\nabla)\boldsymbol{v}. \tag{6}$$

The right hand side gives the change of vorticity due to the action of the force field or due to tilting and stretching of already existing vorticity filament expressed by $(\omega\cdot\nabla)\boldsymbol{v}$.

Creation of vorticity implies that angular momentum is added to the flow. The angular- or moment of momentum balance is satisfied automatically when the Euler equation is satisfied, as shown by e.g. Marshall [12] p. 50. However, an explicit relation between this balance and force fields is not found in literature. In the Appendix it is shown that $\nabla\times\boldsymbol{f}$ expresses, in differential form, the torque applied to a fluid element and similarly (6) the balance of angular momentum. The analysis is restricted to 2D– and 3D axisymmetric flows without swirl.

2.2. Consistency of force field- and kinematical methods

Prandtl [16] showed that a distribution of normal forces acting on a translating lifting surface modelled as a bound vortex sheet γ is equivalent to a pressure distribution at that surface. In the Appendix of van Kuik [9] a similar derivation is presented for a rotating blade. The line of thoughts is the following:

In the kinematical method usually the space occupied by a body is excluded from the flow domain, with appropriate boundary conditions like zero normal velocity applied at the surface. The flow and pressure around it are determined by solving $\rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla p$ resulting in the pressure acting at the surface. In the force field approach this exclusion of the body volume is not made, but the surface is considered as the carrier of $\boldsymbol{f}$ that induces the flow field according to (1). The surface S is considered as a layer of infinitely thin thickness ε, at which $\boldsymbol{f}$ is distributed. After integration of (1) across ε the force term becomes $\boldsymbol{F}$ defined by (5) with $\boldsymbol{f}$ behaving as a Dirac delta function. Integration of the other terms of (1) show that the convective term vanishes for $\varepsilon\to 0$ and the pressure term becomes a pressure jump Δp. In case the surface covers a volume like a wing or rotor blade, the pressure p_0 at the inside is constant, so the result is

$$\boldsymbol{F} = \boldsymbol{e}_\mathrm{n}(p - p_0) \tag{7}$$

with $\boldsymbol{e}_\mathrm{n}$ being the unit vector normal to the surface. For a non-uniform pressure distribution it follows:

$$\nabla\times\boldsymbol{F} = \nabla\times\boldsymbol{e}_\mathrm{n}p \neq 0, \tag{8}$$

so the force field is locally non-conservative and produces vorticity. As an example the pressure field at the surface of the straight wing shown in Fig. 1 is considered. $\nabla\times\boldsymbol{F}$ is integrated along the contour coordinate s of the aerofoil cross section C with $\boldsymbol{F}$ satisfying (7) so $\boldsymbol{F} = \boldsymbol{e}_\mathrm{n}F$. If y denotes the spanwise coordinate the integration concerns $\partial F/\partial s$ and $\partial F/\partial y$. The contribution of $\partial F/\partial s$ vanishes after integration along the closed contour, so:

$$\int_C \nabla\times\boldsymbol{F}\mathrm{d}s = -\boldsymbol{e}_s\int_C \frac{\partial F}{\partial y}\mathrm{d}s. \tag{9}$$

For a wing with a spanwise gradient of the load, the force field is non-conservative. In combination with (5) and (6) this shows that vorticity is produced having a direction in the plane of the cross section known as the trailing vorticity. However, when integrated

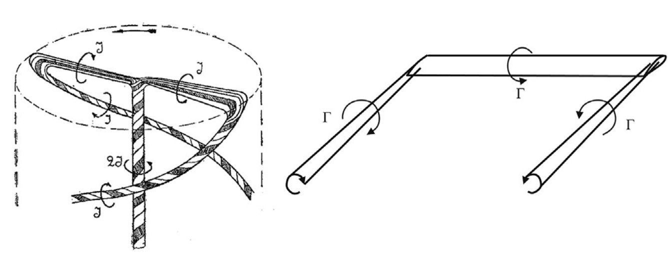

Fig. 1. The rotor model of Joukowsky and wing model of Prandtl.