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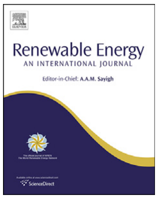

# Pseudo spectral analysis of the energy entrainment in a scaled down wind farm

A. Jensen Newman [a,b,*], Donald A. Drew [b], Luciano Castillo [c]

[a] Texas Tech University, Department of Mechanical Engineering, 2500 Broadway, Box 41021, Lubbock, TX 79409, United States
[b] Rensselaer Polytechnic Institute, Department of Mathematical Sciences, 110 Eighth St., Troy, NY 12180, United States
[c] Texas Tech University, Department of Mechanical Engineering, National Wind Resource Center, Box 41021, Lubbock, TX 79409, United States

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## ABSTRACT

Particle Image Velocimetry from the centerline of a 3 × 5 scaled down array of wind turbines in a wind tunnel was analyzed to gain further understanding of how turbulent transport brings mean kinetic energy (MKE) into the array from the neutrally stable Atmospheric Boundary Layer (ABL) above. Vertical fluxes of MKE due to the Reynolds stresses were computed *i.e.* $-\langle \mathbf{U}\rangle\langle u'v'\rangle$ A modal expansion for $\langle \mathbf{U}\rangle\langle u'v'\rangle$ was constructed based on the Proper Orthogonal Decomposition (POD). By determining each mode's fractional contribution to the total entrainment it was shown that a small number of modes (the first 25) account for 75% of the entrainment. The remaining 25% is achieved asymptotically as the remainder of the modes are included in the representation. Based on this behavior the labels "idiosyncratic" and "asymptotic" were applied to the different mode types. A characteristic wavelength for each mode was defined as the length of a mode's longest positive contribution to the energy entrainment. By this definition it was shown that idiosyncratic and asymptotic modes are characterized by wavelengths greater than and less than $D$ (rotor diameter) respectively so that large percentages of the energy brought into the wind farm are done at scales greater than $D$. Physical reasoning indicates the idiosyncratic modes are associated with larger scale coherent motions whereas the asymptotic modes are associated with small scale turbulent fluctuations. The analysis was repeated for PIV data without turbines. It was shown that the idiosyncratic modes represent the scales which are affected by the presence of the turbines. This further established that the idiosyncratic modes were connected with the larger scales of turbulent motion.

## 1. Introduction

For wind energy to become a prominent source of energy, advanced analysis tools are required to understand the interaction between the Atmospheric Boundary Layer (ABL) and the wind farm. Several approaches to analyze the wake behind the turbines has been considered in the past several decades; ranging from Large Eddy Simulations (LES), (Calaf et al. [8], Meneveau & Meyers [38]); Streamtube theory – including more sophisticated approaches from Lebron et al. [34] and blade element theory [22] (note that streamtube theory depends on blade element theory) which also include techniques based on potential flow theory, (Patel & Katz [42]) which enable numerical computation of the vortex wake behind the wind turbines. For an in depth review of various aerodynamic models for wind turbines see *e.g.* Snell [49]. Recently, the group at Danish Technical University has developed an advanced wake model based on eddy viscosity approach. This method has been proven to properly predict the wake development in full-scale wind farms. A detailed review on all of the methods is given in depth in the article by Newman et al. [40], Sorensen [50], Lebron et al. [34] and Cal et al. [7].

A good summary of the theory of wind turbine wakes is presented by Vermeer et al. [52] and a review of computational work on wakes was published by Sanderse et al. [47]. A general review of computational aerodynamics of wind energy is provided by Sorensen [50]. The Park wake model, originally developed by Jensen [29], and later modified by Katic, Hojstrup, Jensen [30] is one of the most popular analytical wake models used in wind farm modeling. The modified Park wake model and the Eddy Viscosity wake model [1], are other standard wake models. Furthermore, various wake

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[*] Corresponding author. Tel.: +1 7165443184.
*E-mail address:* j.newman@ttu.edu (A.J. Newman).

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**Nomenclature**

| | |
|---|---|
| $U$ | streamwise velocity |
| $u'$ | fluctuating streamwise velocity |
| $V$ | wall-normal velocity |
| $v'$ | fluctuating wall-normal velocity |
| $W$ | spanwise velocity |
| $w'$ | fluctuating spanwise velocity |
| $\langle\rangle$ | time average |
| $D$ | rotor diameter |
| $x$ | streamwise coordinate |
| $y$ | wall normal coordinate |
| $S$ | integration surface |
| $\overline{E}$ | mean kinetic energy, $1/2\langle U_i\rangle\langle U_i\rangle$ |
| $\overline{D}/\overline{D}t$ | mean flow convective derivative, $\partial/\partial t + \langle U_i\rangle\partial/\partial x_i$ |
| $\overline{\mathbf{T}}_i$ | transport of mean kinetic energy, $\left\langle \mathbf{U_j} \right\rangle\langle u_i'u_j'\rangle + \langle \mathbf{U_i}\rangle P/\rho - 2\nu\left\langle \mathbf{U_j} \right\rangle\overline{S}_{ij}$, |
| $\mathcal{P}$ | production, $-\langle u_i'u_j'\rangle\partial\langle \mathbf{U}_i\rangle/\partial x_j$ |
| $P$ | mean pressure |
| $\overline{\varepsilon}$ | dissipation, $2\nu\overline{S}_{ij}\overline{S}_{ij}$, |
| $\overline{S}_{ij}$ | mean strain rate tensor, $1/2(\partial\langle U_i\rangle/\partial x_j + \partial\langle U_j\rangle/\partial x_i)$ |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| $\phi$ | POD mode |
| $\lambda$ | POD eigenvalue |
| $\delta_{ij}$ | Kronecker delta |
| $N_t$ | number of PIV snapshots |
| $a$ | POD mode time coefficient |
| $c$ | eigenvector from method of snapshots |
| $\beta$ | sum of POD eigenvalues |
| $\psi$ | Reynolds shear stress energy entrainment mode |
| $n$ | mode number |
| $\hat{n}$ | unit vector |
| $L$ | characteristic wavelength |
| $R_{ij}$ | two-point correlation tensor |
| $t$ | time |
| MKE | mean kinetic energy |
| $S_t$ | POD analysis domain |
| $C(s)$ | time correlation coefficient |
| $\mathbf{\Lambda}$ | vector of eigenvalues of POD modes |
| $\eta$ | norm of $\mathbf{\Lambda}$ scaled by $\beta$ |
| $E(N_t)$ | relative difference between $\eta^{N_t}$ and $\eta^{\infty}$ |
| $C_p$ | power coefficient |
| $C_t$ | thrust coefficient |
| $\omega$ | vorticity |
| $Q$ | modal contribution to vertical flux of MKE at turbine tip height |

*Subscripts*

| | |
|---|---|
| $i,j$ | velocity component |
| $p$ | time step |
| $s$ | eigenvector component |
| $n$ | mode number |
| $h$ | variable at turbine hub height |
| $\theta$ | azimuthal component of a vector quantity in polar cylindrical coordinates |
| $z$ | axial component of a vector quantity in polar cylindrical coordinates |
| $x$ | indicates a quantity averaged in the streamwise direction |

*Superscripts*

| | |
|---|---|
| $n$ | mode number |
| $*$ | variable at turbine tip height |

models used to analyze full scale wind farms are found in the study by Newman et al. [40] and layout optimization can be found in the studies by Chowdhury et al. [14].

A large portion of previous research using these methods has been focused on the physics of single turbines and aerodynamics of a single blade. The efficiency of a single turbine has also been studied extensively *e.g.* Burton et al. [6] and Snel [49]. There is also presently interest in flow control of wind turbine blades, *e.g.* Wang et al. [53]. Recently, the focus of research has shifted to aerodynamics of an entire wind farms as opposed to single blades and turbines. For example, Frandsen [20] proposed a simple model for the wind speed reduction in the center of a large turbine array and showed good agreement with previously developed empirical relations and computer simulations. Roderick & Coleman [45] conducted numerical simulations of an infinite array and showed a reduction of array efficiency of 30% (*i.e* the ratio of total power output from the array to the power output from a single turbine times the number of turbines in the array) which agreed with predictions made by modeling turbines as roughness. A wind farm optimization study using LES conducted by Meyers and Meneveau [38] showed that separation distance between turbines in the streamwise direction should be approximately 15D (rotor diameters). Dabiri et al. have studied the energy entrainment and power optimization in vertical axis wind farms. They demonstrated that vertical axis turbines have a much smaller wake recovery region than horizontal axis turbines and that vertical flux of energy is a significant source of the turbine's power [31]. Also, by considering an optimized layout they showed an order of increase in power density can potentially be achieved [18].

Development of analytical tools has also been addressed from an experimental point of view recently by performing wind tunnel experiments with scaled turbine arrays. Studies of this nature were performed by Newman et al. [40] and Cal et al. [7] to quantify kinetic energy fluxes due to various turbulent transport terms relative to the power extracted by a turbine, and to establish the effectiveness of equivalent roughness theories proposed by Frandsen [21] and Lettau [35]. Both demonstrated comparable orders of magnitude between fluxes due to Reynolds shear stress and the turbine power measured, and good agreement with Frandsen's roughness theory was observed. More recently, Newman et al. [40] developed a new method of quantifying flow development and used it to show that the mean streamwise velocity becomes fully developed much faster than all other statistics. Further, they showed that in the developing wind turbine boundary layer the flux of energy done by the Reynolds shear stress adds energy from below the wind turbine rotors as well as from above. Chowdhury et al. [15] compared results from a wind farm layout optimization scheme which allowed for different rotor diameters and hub heights in a staggered array with results from a wind tunnel experiment. They found good agreement between the predicted efficiency increases and those observed in the experiment. Specifically, it was shown that a capacity factor increment of 6.4% was possible. Additionally, Zhang et al. [54] used these results to develop an economic model for an optimized wind farm.

Corten et al. [16] performed wind tunnel experiments on a scaled down wind farm to measure profiles of mean velocity and turbulence intensity. It was determined that flow inside the wind farm experiences much less streamwise development than flow