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Modeling large offshore wind farms under different atmospheric stability regimes with the Park wake model

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ABSTRACT

We evaluate a modified version of the Park wake model against power data from a west-east row in the middle of the Horns Rev I offshore wind farm. The evaluation is performed on data classified in four different atmospheric stability conditions, for a narrow wind speed range, and a wide range of westerly wind directions observed at the wind farm. Simulations (post-processed to partly account for the wind direction uncertainty) and observations show good agreement for all stability classes, being the simulations using a stability-dependent wake decay coefficient closer to the data for the last turbines on the row and those using the WAsP recommended value closer to the data for the first turbines. It is generally seen that under stable and unstable atmospheric conditions the power deficits are the highest and lowest, respectively, but the wind conditions under both stability regimes are different. The ensemble average of the simulations does not approach the limits of the infinite wind farm under any stability condition as such averages account for directions misaligned with the row.

1. Introduction

In the last years, investigation of the effect of atmospheric stability on the production of wind farms has gained attention, partly because it has been observed, particularly at large offshore wind farms, that under stable and unstable atmospheric conditions, the wind farms under- and over-perform, respectively, when compared to wind farm data under neutral conditions [6]. Most wake models do not account for stability conditions other than neutral and, thus, model under-performance – when compared to wind farm data – is sometimes attributed to the effect of atmospheric stability.

The Park wake model [8] used in the Wind Atlas Analysis and Application Program (WAsP) [9] is based on the model of Ref. [7], which makes use of the wake decay coefficient k_w to estimate the wind speed reduction for a given thrust coefficient, downstream distance, turbine diameter, and upstream wind speed. It is recommended in WAsP to use $k_w = 0.05$ for offshore wind farms (lower than the recommended value onshore of 0.075). This is because k_w is related to the entrainment of the wake in the atmosphere (it is in fact the slope of the expansion of the wake) and as such it is a function of the surface roughness z_o (the lower the roughness the less wake expansion). Ref. [2], by semi-empirical means, suggested $k_w = 0.5/\ln(h/z_o)$, where h is the turbine's hub height, which generally translates into lower k_w-values than the WAsP recommendations (e.g. Frandsen's k_w becomes 0.039 for a typical wind turbine offshore). Ref. [1] found that using $k_w = 0.03$ adjusted well the results of the Park wake model at the Nysted wind farm when compared to data. Interestingly, at Nysted, i.e. in the South Baltic Sea, stable conditions are mostly observed, whereas at Horns Rev I (a wind farm in the North Sea, where the conditions are generally less stable than at Nysted) good model performance has been found with a slightly higher k_w-value [3].

Here, we present an analysis of wind farm data carried out at the Horns Rev I wind farm, where we are able to classify wind turbine power data into different atmospheric stability classes. A large set of simulations using a modified version of the Park wake model are performed using different k_w-values correspondent to particular atmospheric stability conditions. The simulations are post-processed in order to partly take into account the wind direction uncertainty and compared to the data. Since Horns Rev I is a rather large wind farm, for the wind directions analyzed we might expect that some cases will approach the limits of an infinite wind farm. Therefore, we also present the results of the Park wake model evaluated to its infinite theoretical limits.

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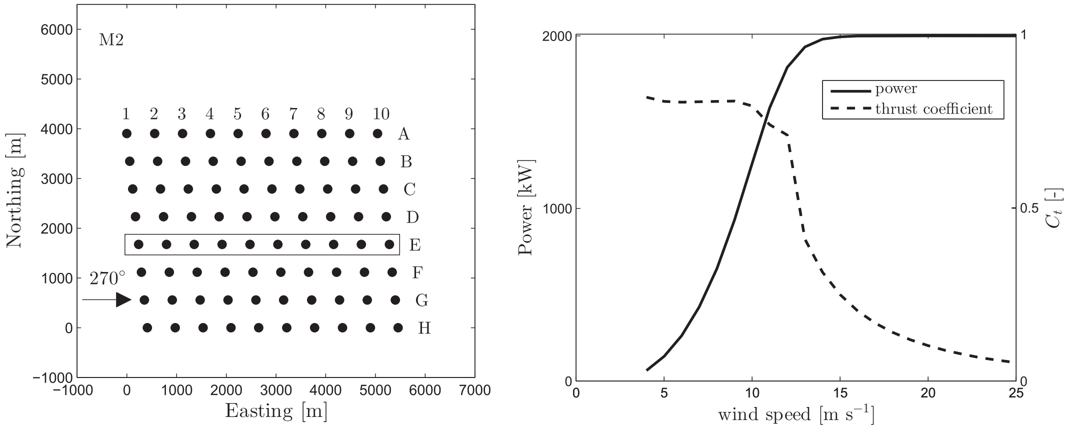

Fig. 1. (Left) The Horns Rev I offshore wind farm and the location of the met mast (M2). Row E (used in this study) is framed. (Right) Power and thrust coefficient (C_t) as a function of wind speed for the Vestas V80 wind turbine.

2. Modified Park wake model

We implemented the Park wake model described in Ref. [8] in a Matlab script to run simulations for a wide variety of wind directions, wind speeds, wind farm layouts, wind turbine specifications, and k_w-values. We refer to it as “modified” because in WAsP the model has been extended to account for the effect of ground-reflected wakes from upwind turbines and our version takes into account the wakes upwind (directly or sideways) only. Partial wakes (from misalignments between the local and the upstream turbines' direction and the wind direction itself) are treated as in Ref. [17], i.e. the local velocity is reduced by a factor depending on the turbine and the wake geometry. Ref. [15] illustrated the modified Park wake model in detail, its approach to account for merging wakes, and the effect of partial wakes.

Ref. [14] showed that adjusting k_w to match the wind speed reductions estimated by a stability-dependent infinite wind-farm boundary-layer model (a totally different model based on the concept of [2], which generally gives higher wind speed reductions in stable compared to unstable conditions) resulted in lower k_w-values under stable compared to unstable conditions. The adjustment was performed evaluating the Park wake model for an infinite wind farm. Similar results were found when evaluating this ‘infinite’ Park wake (IPW) model assuming,

$$k_w \approx u_{*\text{free}}/u_{\text{hfree}} \approx \kappa/[\ln(h/z_o) - \psi_m(h/L)], \tag{1}$$

where $u_{*\text{free}}$ and u_{hfree} are the undisturbed friction velocity and hub-height wind speed, respectively, $\kappa = 0.4$ is the von Kármán constant, and $\psi_m(h/L)$ is the extension to the logarithmic wind profile to account for stability and depends on the height (in this case the hub-height) and atmospheric stability by means of L (the Obukhov length). The expressions for ψ_m can be found in Ref. [10]. Since our Matlab implementation only accounts for upwind wakes, we use the IPW model expressions for the same type of wakes,

$$\delta_\text{I}^2 = \frac{\text{psi}\left[3, 1 + (2s_r k_w)^{-1}\right]}{96 s_r^4 k_w^4}, \tag{2}$$

$$\delta_\text{III}^2 = \frac{-0.0625\text{psi}\left[2, s_f/(s_r k_w)\right]}{s_f s_r^3 k_w^3}, \tag{3}$$

where δ_I^2 and δ_III^2 are the contributions of the wakes directly upwind and upwind partial wakes, respectively, and s_r and s_f are the along- and cross-wind turbine to turbine distances non-dimensionalized by the turbine diameter. The ‘infinite’ limit thus becomes,

$$\frac{u_\infty}{u_\text{free}} = 1 - e_o\left(\delta_\text{I}^2 + \delta_\text{III}^2\right)^{1/2}, \tag{4}$$

where u_∞ is the wind speed upstream the last turbine in the infinite wind farm, u_free the undisturbed wind speed, and $e_o = 1 - (1 - C_t)^{1/2}$, being C_t the thrust coefficient (so it is assumed that this is constant throughout the wind farm). The details of the derivation of the above three equations are given in Ref. [14].

3. Horns Rev I wind farm

The Horns Rev I wind farm is located in the Danish North Sea at about 17 km west from the coast (from the wind farm's northwest corner). A layout of the wind farm showing the positions of the 80 wind turbines (rows are named from A to H and columns from 1 to 10) and a meteorological (met) mast are shown in Fig. 1-left. The turbines are Vestas V80 2 MW machines of 80-m rotor diameter and 70-m hub height. Power and thrust-coefficient curves are illustrated in Fig. 1-right.

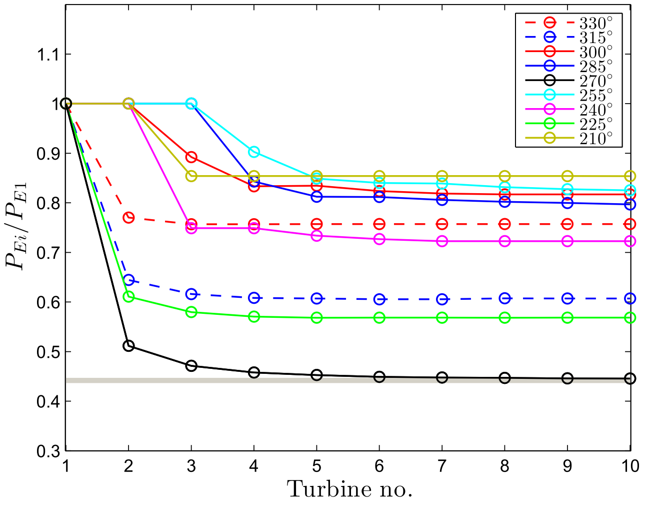

Fig. 2. Simulated power deficits of row E (normalized with the power of turbine 05 P_{E1}) for different westerly directions with $k_w = 0.05$. The thick gray solid line indicates the infinite wind farm limit.