Renewable Energy 68 (2014) 14–24

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Unit-response function for ground heat exchanger with parallel, series or mixed borehole arrangement

D. Marcotte*, P. Pasquier

Civil, Geological and Mining Department, Polytechnique Montréal, C.P. 6079 Succ. Centre-ville, Montréal H3C 3A7, Canada

ARTICLE INFO

Article history:
Received 22 July 2013
Accepted 20 January 2014
Available online 14 February 2014

Keywords:
Finite line source
Parallel arrangement
Series arrangement
Ground heat exchanger

ABSTRACT

A novel approach is presented that allows to predict fluid temperatures entering a Ground Heat Exchanger (GHE) for parallel, series and mixed arrangements of boreholes. The method determines at each time step the heat transfer rates occurring at each borehole so as to reproduce the fluid temperature at the GHE inlet for a specific borehole arrangement. The analytical finite line source model is used to compute the borehole wall temperatures, whereas the fluid temperatures are assumed to vary linearly along the pipes. The method requires to solve a linear system of equations at a small number of time steps. The different systems of equations for each arrangement are determined. A comprehensive 3D finite element numerical model shows good agreement with the computed fluid temperatures. The proposed approach is computationally very efficient. The fluid temperature unit response function can be convolved with any desired heat load to estimate fluid temperatures at the GHE inlet for a wide variety of scenarios.

1. Introduction

Several analytical models are available in the literature [3,10] to predict the temperature response resulting from operation of a ground heat exchanger (GHE). Among the models, the finite line source (FLS) is often used to compute the average temperature along the wall of a borehole heat exchanger (BHE) [5,7,14,17,18]. Fast computation of BHE wall temperature can be done efficiently by spectral methods using either Fast Fourier Transform (FFT) [16] or Laplace transform [11,13]. The FFT approach is faster and easier to apply than the Laplace approach when the heat transfer occurring at each BHE is already known. However, in a GHE, the interactions between the BHEs imply that the heat transfer at each BHE varies in time according to its position in the network, hence, it has to be determined at each time step.

Cimmino et al. [4] and Lazzarotto [15] both used the Laplace transform approach of Lamarche [11] to determine sequentially the heat transfer and the average wall temperature at the BHEs. They used a linear system of equations solved at each time step. Their approach requires the numerical evaluation, at each time step, of an inverse Laplace transform. Pasquier and Marcotte [20,21] proposed a quite different approach where they work simultaneously on all time steps by perturbing iteratively an initial guess heat transfer distribution so as to meet imposed temperature signals on the BHE wall temperatures. Their algorithm converges in a few tens of iterations in all examples tested, and is proven to be exact when the number of iterations reaches the number of time steps.

Neither Cimmino et al. [4], Lazzarotto [15] and Pasquier and Marcotte [20], compared their results to the temperatures obtained with a full 3D numerical model, although Cimmino et al. [4] compared their results to the numerical g-functions of Eskilson [9] and find noticeable differences at long times. Moreover, the previous methods focused on the determination of BHE wall temperature which implicitly assume a parallel arrangement between the BHEs. The examples in Lazzarotto [15] are all with parallel arrangement although Lazzarotto [15] mentions, without providing the pertaining equations, that his approach accommodates parallel and series arrangements.

The objectives of this paper are three-folds: i. adapt the sequential idea of Cimmino et al. [4] and Lazzarotto [15] to the easier to apply FFT spectral approach, ii. develop the unit response functions based on the fluid temperature at the GHE inlet rather than based on the BHE average wall temperature, hence allowing simulation of temperatures for parallel, series and mixed arrangements, and iii. test the proposed approach with a full 3D finite element numerical model.

The paper is structured as follows. The methodological section presents the idea introduced by Lamarche and Beauchamp [13] of splitting the response function in a historical and a contemporary part. Using the FLS and the assumption of linear fluid temperature

* Corresponding author. Tel.: +1 514 340 4711x4620; fax: +1 514 340 3970.
E-mail address: denis.marcotte@polymtl.ca (D. Marcotte).

0960-1481/$ – see front matter
http://dx.doi.org/10.1016/j.renene.2014.01.023

variation along the pipes, the remarkably simple linear systems of equations for different BHE arrangements (parallel, series or mixed) are then presented. The model is applied for a GHE with radial symmetrical distribution of BHE, for parallel and mixed arrangements. The results are compared to those of a full 3D numerical model developed in COMSOL. Computational aspects are discussed to prove the practicality of the sequential FFT approach, even for numerous BHE and long time series. Finally, a detailed numerical example is provided in Appendix A.

2. Methodology

An analytical model (e.g. FLS [5,14]) enables to compute the mean temperature at the borehole wall. Under a steady state hypothesis the mean fluid temperature is obtained as:

$$T_f(t) = (q/H)R_b + \overline{T}_b(t) \tag{1}$$

where q is the heat transferred by the BHE, H is the borehole length, R_b is the BHE equivalent thermal resistance and $\overline{T}_b$ is the average temperature at the BHE wall. Assuming, for simplicity, a linear variation of the fluid temperature along the pipes, one has:

$$T_{in}(t) = T_f(t) + \frac{q}{2\dot{m}C_p}, T_{out}(t) = T_f(t) - \frac{q}{2\dot{m}C_p} \tag{2}$$

where $\dot{m}$ is the fluid mass flow rate and C_p is the specific heat of the fluid.

2.1. Borehole interactions

In a network of n BHEs, the wall temperature of BHE i can be obtained by spatial superposition. Summing the contribution from each BHE, one has:

$$\overline{T}_{bi}(t) = T_0 + \sum_{j=1}^{n} \Delta T_{j\to i}(t) \tag{3}$$

where $\Delta T_{j\to i}(t)$ is the temperature perturbation at BHE i caused by heat emanating from BHE j.

The heat loads are assumed to be a step function with time step Δt. The unit response function has to be calculated at times $m\Delta t$, $m = 1\ldots n_t$. It is convenient to split the temperature perturbations in two terms, the historical part $h_{ij}(m\Delta t)$ due to heat transfer from 0 to $(m-1)\Delta t$ and the present time step contribution $q_j(m\Delta t)f_{\Delta t}(r_{ij})$:

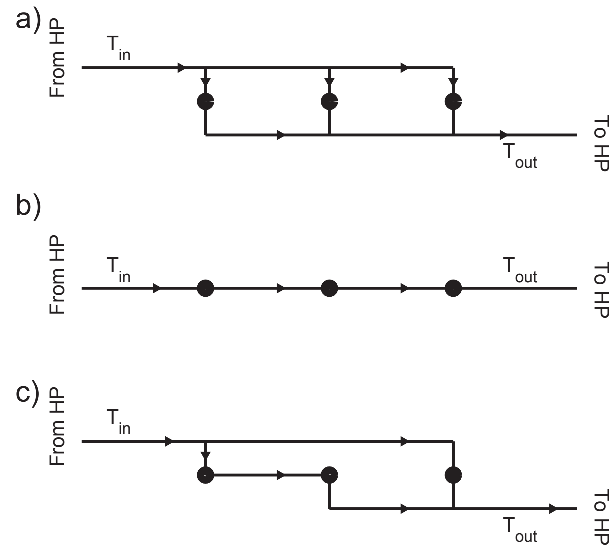

Fig. 1. Different possible BHE arrangements: a) parallel; b) series; c) mixed.

$$\Delta T_{j\to i}(t) = h_{ij}(t) + q_j(t)f_{\Delta t}(r_{ij}) \tag{4}$$

where $f_{\Delta t}(r_{ij})$ is the unit transfer function computed with the analytical model for one time step Δt, and r_{ij} is the distance between boreholes i and j. For $i = j$, one has $r_{ii} = r_b$. Note that the total heat transferred by the n BHEs is $q(t) = \sum_{j=1}^{n} q_j(t)$. The historical part $h_{ij}(m\Delta t)$ can be easily computed by a discrete convolution product:

$$h_{ij}(m\Delta t) = \left(\tilde{\mathbf{q}}_\mathbf{j} * \mathbf{f}\right)(m\Delta t) \tag{5}$$

where, at time $t = m\Delta t$, the step increment vector $\tilde{\mathbf{q}}_\mathbf{j}$ is the vector:

$$\tilde{\mathbf{q}}_\mathbf{j} = \left[q_{j,1}, q_{j,2} - q_{j,1}, \ldots, q_{j,m-1} - q_{j,m-2}, -q_{j,m-1}\right] \tag{6}$$

and the transfer function $\mathbf{f}$ is the analytical model response for the first m time steps under a unit heat load at each BHE evaluated for a distance r_{ij}; $h_{ij}(m\Delta t)$ is obtained as the mth element of the

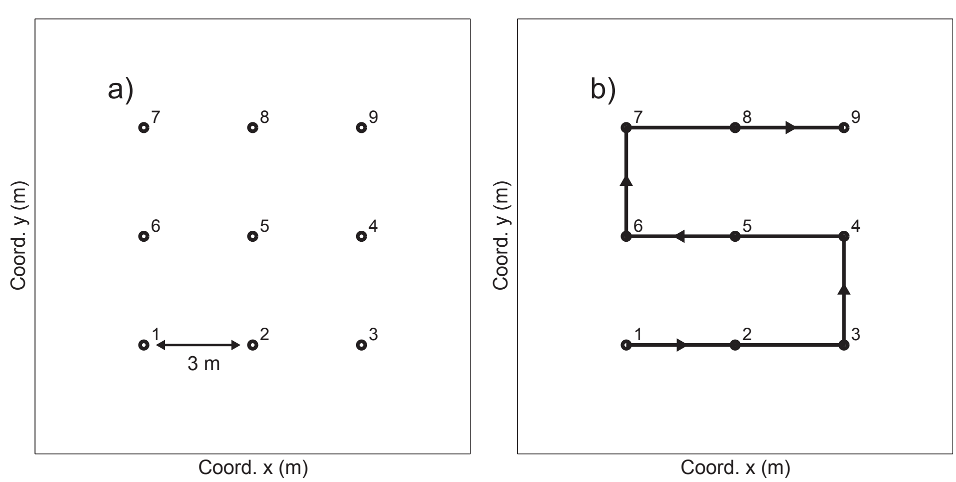

Fig. 2. Location of boreholes and arrangements a) parallel, b) series.