Renewable Energy 68 (2014) 829–841

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Influence of second-order random wave kinematics on the design loads of offshore wind turbine support structures

Anand Natarajan

Department of Wind Energy, Technical University of Denmark (DTU), Risø Campus, 399 Frederiksborgvej, Roskilde 4000, Denmark

ARTICLE INFO

Article history:
Received 11 June 2013
Accepted 28 February 2014
Available online 31 March 2014

Keywords:
Non-Gaussian process
Wave surface kinematics
Gram Charlier series
Convective acceleration
Out of plane loads
Cut-off frequency

ABSTRACT

The impact of wave model nonlinearities on the design loads of wind turbine monopile foundations is delineated based on a second-order nonlinear random wave model that satisfies the boundary conditions at the free surface and by including the effects of convective acceleration in the inertial loads. The second-order nonlinear water kinematics is developed based on a Gram Charlier series expansion using the first four stochastic moments of the wave process. The wave surface velocities and accelerations are expressed using a Taylor series expansion about the mean sea level, which satisfies to the second-order, the unsteady Bernoulli equation and normal flow condition at the free surface. The operating design loads on the monopile are computed using fully coupled and uncoupled methods. The computation of the mud level loads based on the inclusion of nonlinear wave surface kinematics is compared with those obtained when using linear waves and Wheeler stretching. The effect of the spatial derivatives of the wave velocity on the wave surface kinematics is quantified and shown to determine the wave spectral cut-off frequency limit. The spatial derivatives of wave velocity also participate in the expression for the wave convective acceleration, whose effect is demonstrated on the inertial loads on the foundation in the presence of ocean currents. The effect of nonlinear water kinematics on the monopile design load reveals the large frequency bandwidth of wave structure interaction, but the phase differences between the hydrodynamic loads with the rotor loads tend to lower the probability of joint simultaneous extreme peaks in hydrodynamics and rotor loads.

1. Introduction

With offshore wind turbines moving to 30 m–50 m water depths in the near future, the hydrodynamic loads at these moderate water depths can be significant in comparison to the rotor loads, which implies the hydrodynamic models may require the use of more accurate water kinematics than Airy linear wave theory or nonlinear regular wave models [1], as presently used in design loads simulation. Since the free surface boundary conditions are applicable on a wave surface and not at the mean sea level (MSL), the water kinematics is nonlinear. Only under the conditions that the wave height is negligible with respect to the wave length and water depth, this nonlinearity may be neglected. Most wind turbine load simulation codes employ geometric stretching to map the region between the mean sea level and the wave free surface back to a domain below the mean sea level. Wheeler stretching [2] is a popular stretching method used in wind turbine loads predictions, but the water kinematics obtained using Wheeler stretching can significantly under predict or over predict wave velocities [3], which in turn affect the design loads. A key reason for this is that Wheeler stretching does not satisfy the Laplace equation for potential flow, the error of which is negligible in deep waters, but can be significant at moderate water depths of 35 m. Therefore at moderate water depths, the wave models must include the wave surface boundary conditions, which are nonlinear.

A key benefit of using nonlinear random wave models is that the simulated process can be non-Gaussian and the effect of higher order stochastic moments of the wave process on design loads can be determined. This enables taller wave crests to be simulated at a given significant wave height than when using linear Gaussian waves, an aspect that has been observed in measurements [4]. Various methods of deriving nonlinear random ocean wave models have been put forth, such as through Boussinesq equations [5] which compute perturbation expansions of the velocity potential of several orders about a known vertical station. The Boussinesq equations are accurate for shallow waters such as for analysis of shoaling effects, but these equations may require quartic and higher terms if wave crests are described at moderate water depths. On the other hand, second-order nonlinear random wave mechanics have been described at length by various authors [6–8] and have been used in modeling

E-mail address: anat@dtu.dk.

http://dx.doi.org/10.1016/j.renene.2014.02.052
0960-1481/

waves for more than 30 years. However second-order nonlinear random wave models have not found wide application in wind turbine design codes since most offshore wind turbine design loads at shallow water depths below 20 m are dominated by the rotor loads. Langley [8] describes a computationally efficient formulation for the second-order random wave elevation and this has been extended by Moan to wave kinematics [9]. The nonlinear wave particle velocity and acceleration for a second-order random wave is expressed as

$$\begin{aligned}
u &= \sum_{i=1}^{N}\frac{gk_i}{\omega_i}A_i\frac{\cosh(k_i(z+h))}{\cosh(k_ih)}\cos(k_ix-\omega_it+\beta_i)+\sum_{i=1}^{N}\sum_{j=1}^{N}B_{uj}B_{ui}\left[P_{uij}\cos\left(k_{ij}^{-}x-\omega_{ij}^{-}t+\beta_i\right)+Q_{uij}\cos\left(k_{ij}^{+}x-\omega_{ij}^{+}t+\beta_i\right)\right]\\
w &= \sum_{i=1}^{N}\frac{gk_i}{\omega_i}A_i\frac{\sinh(k_i(z+h))}{\cosh(k_ih)}\sin(k_ix-\omega_it+\beta_i)+\sum_{i=1}^{N}\sum_{j=1}^{N}B_{wj}B_{wi}\left[P_{wij}\sin\left(k_{ij}^{-}x-\omega_{ij}^{-}t+\beta_i\right)+Q_{wij}\sin\left(k_{ij}^{+}x-\omega_{ij}^{+}t+\beta_i\right)\right]\\
a &= \sum_{i=1}^{N}-gk_iA_i\frac{\cosh(k_i(z+h))}{\cosh(k_ih)}\sin(k_ix-\omega_it+\beta_i)+\sum_{i=1}^{N}\sum_{j=1}^{N}C_{aj}C_{ai}\left[R_{ij}\sin\left(k_{ij}^{-}x-\omega_{ij}^{-}t+\beta_i\right)+S_{ij}\sin\left(k_{ij}^{+}x-\omega_{ij}^{+}t+\beta_i\right)\right]\\
a_w &= \sum_{i=1}^{N}gk_iA_i\frac{\sinh(k_i(z+h))}{\cosh(k_ih)}\cos(k_ix-\omega_it+\beta_i)+\sum_{i=1}^{N}\sum_{j=1}^{N}C_{wj}C_{wi}\left[R_{ij}\cos\left(k_{ij}^{-}x-\omega_{ij}^{-}t+\beta_i\right)+S_{ij}\cos\left(k_{ij}^{+}x-\omega_{ij}^{+}t+\beta_i\right)\right]
\end{aligned}\tag{1}$$

where B,C, P_{ij}, Q_{ij}, R_{ij}, S_{ij}, are terms dependent on the wave amplitude, frequency and wave number and require a fairly detailed formulation, which is provided in Ref. [6]. The superscripts + and − refer to a summation or difference between the frequencies, ω_i, ω_j or between the wave numbers k_i, k_j, g is the acceleration due to gravity, A_i is the linear wave amplitude and h is the water depth.

The mud level moments and forces at the base of the foundation play a crucial role in foundation design. The influence of the nonlinear waves on the mud level moments and forces is to be evaluated in both fatigue and ultimate load situations. For nonlinear waves in moderate water depths, it is realistic to utilize the wave surface kinematics directly computed based on the moments of the stochastic process and by satisfying the free surface boundary conditions. The Morison equation [10] is normally used to compute the hydrodynamic loads on wind turbine support structures, so long as the dimension of the support structure is considered small (<20%) with respect to the wave length of the wave. The local fluid acceleration is conventionally used as described in many standards [10] to compute the hydrodynamic inertial loading on offshore structures. However, the convective acceleration of the fluid is also a factor that needs to be assessed. Thus the usage of the total derivative of acceleration as opposed to the partial derivative in the Morison equation may result in increased inertial loading above the MSL, due to the convective acceleration and its interaction with ocean currents. This effect can be readily investigated through the use of a nonlinear wave model. Since wind turbine support structure design relies greatly on simulation models and an understanding of site specific conditions, the work herein is focused on bringing forth potential design scenarios that reflect the impact of the wave models and the differences in the simulated design loads thereby achieved.

2. Stochastic analysis of nonlinear waves

Since Eq. (1) involved a double Fourier summation, it can be time consuming to implement this at every vertical station along the water depth where the water kinematics is to be evaluated for long time intervals. A computationally simpler technique is sought whereby it is not required to evaluate double Fourier summations. Following the method of Winterstein [11], a Gram-Charlier series may be used to approximate the second-order nonlinear wave kinematics by using the first 4 moments of the stochastic wave process. This requires that the moments of the wave stochastic process be computed either from simulations or from experimental observations. The moments can be readily derived, if the stochastic wave process $x(t)$ can be expressed as

$$x(t) = \sum_{j=1}^{2N}\left(\beta_jb_j(t)+\lambda_jb_j^2(t)\right)\tag{2}$$

where, β_j is the linear wave response parameter transformed to a modal space, λ_j are the eigenvalues of the second-order coefficient matrices, such as determined from Eq. (1) and b_j are standardized Gaussian parameters expressed in modal space. Details of the derivation of Eq. (2) for expressing wave elevation or wave kinematics can be found in Refs. [8,9] and the nonlinear terms in Eq. (2) are computed directly from Eq. (1) by reformulating the coefficients in Eq. (1) into a matrix representation that allows an eigenvalue analysis, details of which can be referred in Ref. [9]. Since the wave kinematic variable $x(t)$ is expressed as a polynomial function in Eq. (2), its stochastic moments are directly derived. The first four moments of this second-order wave process is expressed in Eq. (3) as:

$$\begin{aligned}
m_x &= \sum_{j=1}^{2N}\lambda_j\\
\sigma_x^2 &= \sum_{j=1}^{2N}\left(\beta_j^2+2\lambda_j^2\right)\\
\alpha_{3x} &= \frac{1}{\sigma_x^3}\sum_{j=1}^{2N}\left(6\beta_j^2\lambda_j+8\lambda_j^3\right)\\
\alpha_{4x} &= 3+\frac{1}{\sigma_x^4}\sum_{j=1}^{2N}\left(48\beta_j^2\lambda_j^2+48\lambda_j^4\right)
\end{aligned}\tag{3}$$

where α_{3x} and α_{4x} are the skewness and kurtosis of the wave process, $x(t)$. It is now possible to express $x(t)$ using these stochastic moments derived in Eq. (3) by modulating a Gaussian process using Hermite coefficients that are derived from these moments. The Hermite coefficients are orthogonal with respect to the Gaussian weighting function and such polynomial expansions have been proven to be convergent [11]. This implies that the quantity of