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Technical note

Short-term behavior of classical analytic solutions for the design of ground-source heat pumps

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ABSTRACT

Over the years several methods have been proposed to simulate and design the earth heat exchanger for a ground-source heat pump (GSHP) system. Some of these methods are based on numerical techniques while others rely on analytic solutions. Among the latter, two classical solutions have been extensively used over the years, the infinite line source (ILS) solution and the infinite cylindrical source (ICS). These solutions were known to overestimate the fluid temperature when the time scale is important and are valid only in a time range between a minimum and a maximum value which are often adequate for must design applications. It is usually accepted that for small Fourier numbers, the ICS solution should be used instead of the ILS. This paper revisits the short-term behavior of these solutions and we arrive at different conclusions than those usually accepted in the literature if the Fourier number is based on the borehole radius, which is normally the case. The reasons for these discrepancies are discussed and several options are proposed.

1. Introduction

A typical vertical borehole being part of a ground-source heat pump (GSHP) system is shown in Fig. 1. A plastic tube made of high density polyethylene (HDPE) wherein circulates water or water with antifreeze, pumps or rejects heat into the ground. A grout is normally used to seal the heat exchanger. In most systems, several boreholes are used. A typical design of the earth heat exchanger calculates the total length of the bore field so that the exit temperature of the fluid is within acceptable range for the good operation of the heat pump. This calculation needs the properties of the soil, the geometry of the borehole, the amount of heat exchanged with the ground during the time (several years) of the system operation. In order to evaluate the temperature at the exit of the earth heat exchanger we have to evaluate the transient heat transfer in the ground. Several methods were proposed over the years to perform this step. Some authors rely on numerical techniques and others propose the use of analytical solutions. We will not discuss here the merits of each approaches, both having their advantages and inconvenients. Let's just say that analytical methods are still widely used especially when a quick (but accurate) design is needed. Among those analytical methods, the two most widely used are the infinite line source (ILS) and the infinite cylindrical source (ICS) solutions. The long-term behavior of those two solutions is identical and both overestimate the temperatures of the fluid [1]. In that case other solutions are suggested like the finite line source solution [2–4]. The time range for this to happen is however several years of operation and in this paper we are only concerned with the short-term behavior so we will not discuss this point at all.

For the short-term behavior, which is in the order of hours for a typical soil, both solutions neglect the internal capacity of the borehole and for that are known to lack accuracy. This is the reason why this subject has been an active part of research in recent years [5–13]. Yavuzturk [5] was one of the first researchers to study the effect of short-time behavior on a typical design where he used a numerical model to generate what he called, a short-time *g-function*. Analytical methods have been proposed by Young [6], Sutton et al. [7], Lamarche and Beauchamp [8] and Javed and Claesson [9]. Network based methods were proposed among others by Bauer et al. [10], De Carli et al. [11] Zarella et al. [12] and Pasquier and Marcotte [13]. Those approaches have all their own merits and drawbacks and any one of those will be a better choice to analyze the short-time behavior than the ILS or the ICS solutions. Nevertheless, some may find them difficult to implement and maybe too cumbersome to use for quick length estimation so it might be interesting to reanalyze the behavior of the two classical solutions and this is the purpose of this paper. In the next section we resume

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Nomenclature

D	diameter (m)
E_1	exponential integral function
F_o	Fourier number, defined in (8,11)
G	G-function
h	convection coefficient (W m^{-2} K^{-1})
H	step response
J	Bessel function of the first kind
k	thermal conductivity (W m^{-1} K^{-1})
L	borehole height (m)
q	heat load (W)
q'	heat flux per unit length (W m^{-1})
r	radial coordinate (m)
R'	unit length thermal resistance (K m W^{-1})
t	time (s)
T	temperature (K)
Y	Bessel function of the second kind
z	dummy integration variable

Greek letters

α	thermal diffusivity (m^2 s^{-1})
θ	angular variable (rd)

Subscripts

b	at the borehole radius
pi	at the inside pipe radius
po	at the outside pipe radius
f	fluid
o	far-field value

the theory behind the two solutions and explain the reasons why they lack accuracy in the short-time behavior. In Section 2, comparisons are made between those solutions and a numerical simulation done with finite element commercial software which serves as a reference and the short-time analytical solution proposed by the author [8]. The conclusions we arrive are different than the one usually accepted in the literature [14] and the reasons for that will be explained in Sections 3 and 4. Finally the impact on a typical design will be analyzed in Section 5. It is important to remind that the goal of this paper is not to propose a new method for the design of the ground heat exchanger and neither to judge the existing ones. The main objective is to provide a contribution to the dilemma regarding the use of the ILS and ICS whenever these solutions are used.

2. The infinite line source and the infinite cylindrical source solutions

The heat transfer between the calorimetric fluid and the ground in a typical borehole is a three dimensional transient heat transfer problem. Due to the slenderness of the borehole, the axial effect can be neglected for the time range that we want to analyze. In that case, the thermal system shown in Fig. 2 can be modeled by the following equations:

$$\frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} = \frac{1}{r^2}\frac{\partial^2 T_i}{\partial \theta^2} + \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r}, \quad i = 1,2,3(\text{plastic}, \ \text{grout}, \ \text{soil}) \tag{1}$$

$$T(r,\theta,0) = T_o, \quad -k\frac{\partial T}{\partial r}\bigg|_{r=r_{pi}} = h\Big(T_f - T\Big)\Big|_{r=r_{pi}} = q''_b(t) = \frac{q'_b(t)}{4\pi r_{pi}} \tag{2}$$

From now on, the heat flux is assumed positive when heat is rejected into the ground. Even though we neglect the axial dependence the preceding problem does not have an analytical solution and simplification is needed to find one. Two approaches are used to solve the problem analytically. In the first one, a constant step heat flux is assumed to come from a point source at the origin (Fig. 3a). This leads to the infinite line source solution which solution is found to be:

$$T(r,t) - T_o = \frac{q'_b}{4\pi k_{soil}} \int_{r^2/4\alpha t}^{\infty} \frac{e^{-u}}{u} du = \frac{q'_o}{4\pi k_{soil}} E_1(r/(4\alpha t)) \tag{3}$$

where E_1 is the exponential integral function. The second solution assumes that the constant heat flux is imposed at the borehole radius. This solution is known to be the infinite cylindrical source. Both solutions are discussed in the classical book of Carslaw and Jaeger [15].

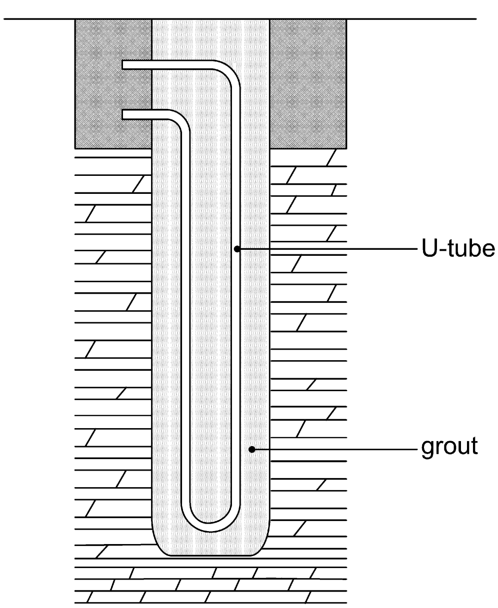

Fig. 1. Schematic of the borehole.

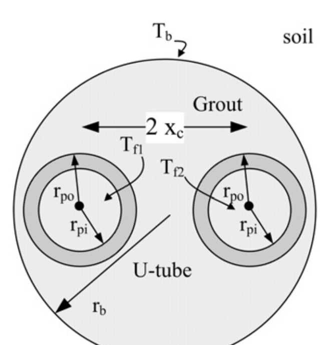

Fig. 2. Cross-section of the borehole.