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A note on multiple reflections of radiation within CPCs and its effect on calculations of energy collection

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ABSTRACT

In this article, multi-reflection of radiation on both parabolic reflectors of CPC with one sided flat absorber and its effect on calculations of energy collection are theoretically investigated. Results show that, when a beam of radiation strikes on the aperture at incidence angle near zero, a considerable fraction of radiation will arrive on the absorber after more than two reflections, but the fraction of radiation that arrive the absorber after more than three reflections is considerably small. Comparative analysis of annual radiation collected by east-west oriented CPC based different reflection models indicates that the two-reflection model where more than three reflections are not considered can accurately estimate the annual collectible radiation, and for the case of CPCs with high reflectivity such as larger than 0.8 and larger acceptance angle such as larger than 20°, the one-refection model where more than two reflections are not considered is acceptable to estimate annual radiation collected by CPC.

1. Introduction

CPC, independently suggested by Hinterberger and Winston of United States, Ploke of Germany, Baranov and Melnikov of Russia in 1966 [1], is a typical non-imaging ideal concentrator. In recent years, CPCs have been widely tested for concentrating solar radiation on solar cells to increase the power output of photovoltaic systems and reduce the cost of electricity generation [2–5].

For such applications of a CPC, the first concern for engineers is that how much radiation could be annually received by the absorber to which solar cells are attached so as to ascertain whether such system is attractive economically. Previous study of the authors showed that the optimal acceptance half-angles of fixed east-west oriented CPCs (EW-CPCs) were in between 25 and 26°, and the maximum annual average optical concentration factors, depending on site climatic conditions, were in between 1.4 and 1.7 [6]. However, the optical loss due to imperfect reflectors of CPCs was not considered in the work. A study performed by Rabl based on ray-tracing shows that when solar rays enter CPC at the angle nearly perpendicular to the aperture, a fraction of radiation that reaches the absorber undergoes more reflections on both reflectors of CPC, and dependence of average reflection number of solar rays within full and truncated CPC on the acceptance angle are graphically presented [1]. But the quantitative effect of more reflections on calculations of energy collection of CPCs was not studied.

In this work, fractions of radiation that arrives on the absorber of CPCs with flat one-sided absorber after more than any number of reflections were theoretically investigated, and effects of multiple reflections on calculations of annual energy collection of fixed EW-CPC troughs (1T-EW-CPC) and those with the tilt-angle of the aperture being yearly adjusted four times at three tilts (3T-EW-CPC) with the aim to determine whether multiple reflections within CPCs must be considered in calculations of energy collection of CPCs.

2. Mathematical methodology for the analysis of multiple reflections

2.1. Optical efficiency factor and one-reflection model

For full CPC with flat one-sided absorber (the width of absorber is assumed to be 1 for convenient analysis), the equation of parabolic reflector (right reflector) in the suggested coordinate system (see Fig. 1) can be expressed by:

$$\begin{cases} x = \dfrac{(1+\sin\theta_a)\sin\theta}{1-\cos(\theta+\theta_a)} - 0.5 \\ y = \dfrac{(1+\sin\theta_a)\cos\theta}{1-\cos(\theta+\theta_a)} \end{cases} \quad (\theta_a \le \theta \le 0.5\pi) \tag{1}$$

where θ_a is the acceptance half-angle, and θ is the polar angle of any point P on the parabolic reflector.

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Multi-reflection takes place on the same reflector of both reflectors as a result of the fact that the tilt-angle of tangent line passing through any point on parabolic reflectors gradually decreases from the top end (A or A′) to the lower end (B or B′). Thus when solar rays incidence on the aperture at projection incidence angle θ_p less than θ_f, the entire absorber will be directly irradiated, and a fraction of radiation will arrive the absorber after more reflections on both reflectors, as shown in Figs.1–3. For the case of $\theta_f \le \theta_p \le \theta_a$, a fraction of radiation incident on the aperture will directly reaches on the absorber, and the rest will arrive on the absorber after just one reflection. For full CPCs, θ_f is given by:

$$\tan\theta_f = \frac{C_g - 1}{C_g + 1}\tan\theta_a \tag{2}$$

where C_g is the geometrical concentration factor of full CPC, and is expressed by:

$$C_g = 1/\sin\theta_a \tag{3}$$

To be convenient for theoretical calculations of radiation that actually arrives on the absorber of CPC, the optical efficiency factor is introduced and defined as the fraction of radiation incident on the aperture that reaches on the absorber directly or indirectly by reflections. For a given CPC, the optical efficiency factor, $f(\theta_p)$, is the function of projection incidence angle (θ_p) (referred to as incidence angle in this paper). In this work, reflectors of CPCs are assumed to be perfect specular reflectors with the reflectivity of ρ, thus, $f(\theta_p)$ can be expressed by:

$$f(\theta_p) = \begin{cases} f_n & |\theta_p| < \theta_f \\ \rho + 0.5(1-\rho)(1+\sin\theta_a)(1-\tan|\theta_p|/\tan\theta_a) & \theta_f \le |\theta_p| \le \theta_a \\ 0 & |\theta_p| > \theta_a \end{cases} \tag{4}$$

where f_n is the optical efficiency factor for the case of $|\theta_p|\langle\theta_f$ calculated based on the model where more than n-reflections is considered. In the case of more than two reflections is not considered (referred to as the one-reflection model), namely, assumed that all radiation will either directly arrive on the absorber or arrive on the absorber after only one reflection in the case of $|\theta_p|\langle\theta_f$, the optical efficiency factor, f_1, is simply expressed by:

$$f_1 = 1/C_g + (1 - 1/C_g)\rho \quad \left(|\theta_p| < \theta_f\right) \tag{5}$$

2.2. Two-reflection model

In this model, more than three reflections is not considered, and the radiation arriving on the absorber after more than two reflections is simply regarded to be that arrives on the absorber after only two reflections (referred to as the two-reflection model). As shown in Fig. 2, solar rays incident on AM of the right parabola (orA′M′ of the left parabola) will undergo more than two reflections before arriving the absorber, whereas solar rays incident on BM (B′M′) will arrive the absorber after one reflection, and M (M′) is the critical point from which solar ray reflected just strikes at the end B (B′) of the absorber. This means that when solar rays enter CPCs at $\theta_p < \theta_f$, a fraction, η_2=(AD + D′A′)/AA′, of radiation incident on the aperture would arrive on the absorber after more than two reflections. The calculation of η_2 is presented as follows.

The slop angle of tangent line passing through point M (see Fig. 2) can be obtained based on Eq. (1) as follow:

$$\tan\beta_m = \frac{dy}{dx} = \frac{\sin\theta_a + \sin\theta_m}{\cos\theta_a - \cos\theta_m} \tag{6}$$

where θ_m is the polar angle of point M. In turn, given β_m, θ_m can be calculated based on above equation as follows:

$$\sin\theta_m = \cos^2\beta_m\left(a + \tan\beta_m\sqrt{1/\cos^2\beta_m - a^2}\right) \tag{7}$$

$$\text{and } a = \tan\beta_m\cos\theta_a - \sin\theta_a \tag{8}$$

Base on the reflection law of light, one has:

$$\beta_m = 0.25\pi + 0.5(\alpha + \theta_p) \tag{9}$$

and α is given by:

$$\tan\alpha = \frac{y_m}{x_m - 0.5} = \frac{(1+\sin\theta_a)\cos\theta_m}{(1+\sin\theta_a)\sin\theta_m + \cos(\theta_a+\theta_m) - 1} \tag{10}$$

where x_m and y_m are the x and y coordinate component of point M, respectively. Given θ_a and θ_p, one assigns an initial value (such as θ_a) to θ_m, then obtains α based on Eq. (10) and β_m based on Eq. (9), after then a new θ_m can be found based on Eq. (7), and the true θ_m can be finally obtained by repeating above calculations until the difference of θ_m between two calculations less than a specified value (such as 0.00001). On knowing θ_m, DA can be calculated by:

$$DA = 0.5C_g - x_m + (h - y_m)\tan\theta_p \tag{11}$$

Where h is the height of CPC, and calculated by:

$$h = 0.5(1 + C_g)/\tan\theta_a \tag{12}$$

Similarly, $\theta_{m'}$, the polar angle of point M′ on the left parabola of CPCs can be obtained based on the same method as finding θ_m by setting $\theta_p = -\theta_p$, then D′A′ can be calculated by:

$$D'A' = 0.5C_g - |x_{m'}| - (h - y_{m'})\tan|\theta_p| \tag{13}$$

Knowing DA and D′A′, the fraction of radiation incident on the aperture that reaches on the absorber of CPCs after more than two reflections is calculated by:

$$\eta_2 = (DA + D'A')/C_g \tag{14}$$

Knowing η_2, the optical efficiency factor, f_2, based on the two-reflection model is calculated by:

$$f_2 = 1/C_g + \eta_2\rho^2 + (1 - \eta_2 - 1/C_g)\rho \quad \left(|\theta_p| < \theta_f\right) \tag{15}$$