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Effect of the wake behind wind rotor on optimum energy output of wind farms

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ABSTRACT

This study deals with the modeling of the wake effect on the energy extracted from the wind farms. It covers the wake effect of the interaction of the upstream wind rotor with/without the upstream right and/or upstream left wind rotor. A mathematical model representing a single wake model based on the linear description of the wake is developed in order to predict the wind speed inside the wake region at any downstream distance within the wind farm. Two different types of turbines with diameters of 62 m and 100 m are considered. Accordingly the effect of the wake on the energy produced from the wind farms is estimated.

A number of different wind farm layouts are studied. Case studies including 3 × 3, 4 × 4, 6 × 6, 1 × 16, 16 × 1, 2 × 8, and 8 × 2 layouts are considered. Extracted energy is calculated in each case and an optimum layout is determined from different layouts. The effectiveness of the other layouts with respect to the optimum is obtained. The results showed that there is a drop in the annual extracted energy from the above mentioned layouts depending on the W.T. distances separating the W.T.'s. The wind speed was assumed to be 15 m/s with 10*D* downstream distances. The losses are estimated to be 20% for 3 × 3 (rows × column), 32% for 4 × 4, 46% for 6 × 6, 12.8% for 16 × 1, 23.3% for 2 × 8, and 29% for 8 × 2 when these layouts are compared to 1 × 16 layout as an optimum layout.

1. Introduction

A wind farm contains a number of horizontal wind turbines, these W.T.s are positioned aligned in clusters facing the wind direction, they are eclectically connected together in one place depending on the available area of the project, and the quantity of energy required [2].

Each wind rotor generates a turbulent region called wake. This wake causes a sudden decrease in velocity, consequence it causes a decrease in the quantity of air and wind speed entering the downstream turbine, so that less energy will be produced by the downstream turbine. As air comes out of the wind turbine rotor, its initial diameter is almost equals to the diameter of the turbine rotor. Then it tends to spread out conically.

The objective of this study is to estimate the effect of wake on energy extracted from a wind farm. This paper deals with a mathematical model for the effect of wake, which uses only the commonly available parameters and data of the wind farm and the turbines within the farm.

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2. Wake wind speed (v_w)

The radius of the cone can be represented as a function of the downstream distance from the turbine location as follows;

$$r(x) = r_{\mathrm{rot}} + x^{*}\tan\alpha \tag{1}$$

where $r(x)$ is radius of the shadow cone, r_{rot} is radius of upstream turbine, x is the distance between the turbines, and tan α is the factor of the cone. The factor tan α was found, to have two values; 0.04 for turbines facing the wind stream in the first row of the wind farm, and 0.08 for turbines downstream [3].

Corresponding to the following assumptions; steady and one dimensional incompressible flow with the basis of the principle of mass conservation, the wake wind speed can be computed at any downstream distance. Referring to Fig. 1 and to the integral continuity equation, at any location along the downstream distances, the mass flow rate is given by;

$$\frac{\partial m}{\partial t} = A(x)^{*}v_w(x)^{*}\rho \tag{2}$$

Also, at any location along the downstream distance, the sweeping wind speed has two different values, one inside the shadow cone (v_{w0}), and the other outside (v_0). This leads to the equation of the mass flow rate as;

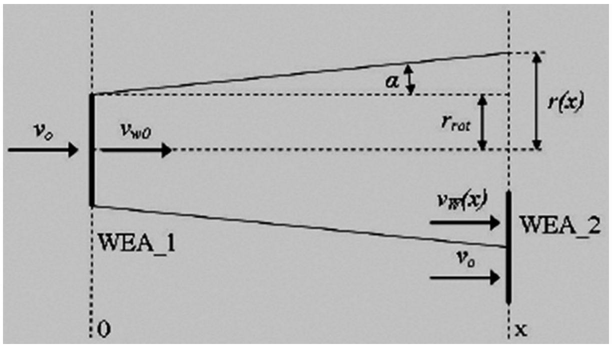

Fig. 1. Turbine and shadow cone [5].

$$\frac{\partial m}{\partial t} = A_{\text{rot}} {*} v_{w0} {*} \rho + (A(x) - A_{\text{rot}}) {*} v_0 {*} \rho \tag{3}$$

By equating equations (2) and (3), the wake speed at the downstream turbine location as a function of the free wind speed is given by the following equation;

$$v_w(x) = v_0 + (v_{w0} - v_0) {*} \left(\frac{r_{\text{rot}}}{r(x)}\right)^2 \tag{4}$$

where $v_w(x)$ is the wake wind speed at distance x, v_0 is a free stream speed, v_{w0} is the wind speed behind the rotor, r_{rot} is the radius of the rotor, and $r(x)$ is the radius of the wake region a long distance x.

The calculation of wake effect from neighboring turbines is determined by assuming the square velocity deficit of a mixing wake to be equal to the sum of squared velocity deficit for each wake at the calculated downstream distance [4].

$$(v_0 - v)^2 = (v_0 - v_w)_1^2 + (v_0 - v_w)_2^2 + \ldots. \tag{5}$$

The resultant wind speed over the rotor at distance x is:

$$v = v_0 - \sqrt{\sum\nolimits_{\text{for all wakes}}^{n} (v_0 - v_w)^2} \tag{6}$$

3. Power output

The power associated with a mass flow rate entering the cross-sectional area of the rotor, that is a function of free wind speed is given by the equation;

$$P_0 = \frac{1}{2} {*} \frac{\partial m}{\partial t} {*} v_0^2 \tag{7}$$

The theoretical maximum extractable power is given by the equation;

$$P_{\text{mech\_th}} = \frac{1}{2} {*} \frac{\partial m}{\partial t} {*} \left(v_0^2 - v_{w0}^2\right) \tag{8}$$

The theoretical power coefficient is given as follows [1];

$$C_{P_{\text{th}}} = \frac{P_{\text{th}}}{P_0} = \frac{1}{2} {*} \left(1 + \frac{v_{w0}}{v_0}\right)\left(1 - \frac{v_{w0}^2}{v_0^2}\right) \tag{9}$$

4. Wind speed behind the rotor (v_{w0})

Referring to Betz limit, the physical possible amount of power extracted is not more than 59.3% of the power in the wind, and the minimum possible power is being zero. Thus it can be considered that the values of $C_{P_{\text{th}}}$ lie within the range of $0 \le C_{P_{\text{th}}} \le C_{P\_\text{Betz}}$. The important equation (9) which is the function of $C_{P_{\text{th}}}$ has different set of solutions.

These can be represented as follows [5];

$$\frac{v_{w0}}{v_0} = \frac{4{*}\cos(\varphi/3) - 1}{3} \quad \text{for } C_{P\_\text{th}} < \frac{8}{27} \tag{10}$$

Or

$$\frac{v_{w0}}{v_0} = -\frac{4{*}\cos(\theta/3) + 1}{3} \quad \text{for } C_{P\_\text{th}} \ge \frac{8}{27} \tag{11}$$

With

$$\text{And} \quad \begin{aligned} \varphi &= \cos^{-1}\left(1 - \frac{27}{8} {*} C_{P_{\text{th}}}\right) \\ \theta &= \cos^{-1}\left(\frac{27}{8} {*} C_{P_{\text{th}}} - 1\right) \end{aligned} \tag{12}$$

By these equations (10)–(12), the relationship between the ratio of the wind speed behind the turbine to the speed in front of the turbine and the ideal power coefficient $C_{P_{\text{th}}}$ is established. Normally, the turbine reaches its maximum efficiency at its design tip speed ratio thus,

$$\eta = \frac{C_{P_{\max}}}{C_{P_{\text{opt}}}} \tag{13}$$

where $C_{P_{\text{opt}}}$ represents the optimum power coefficient which can be considered as a measure for the approximation of the power losses. The optimum coefficient is only a function of the tip speed ratio and can be written as [5];

$$C_{P_{\text{opt}}} = \frac{16}{27} {*} \left(1 - \frac{0.219}{\lambda^2} - \frac{0.106}{\lambda^4} - \frac{2}{9} {*} \frac{\ln\lambda^2}{\lambda^2}\right) \tag{14}$$

where λ is the tip speed ratio.

The theoretical power coefficient $C_{P_{\text{th}}}$ can be obtained from the following relationship;

$$C_{P_{\text{tg}}} = C_P {*} \left(C_{P_{\text{opt}}} / C_{P_{\max}}\right) \tag{15}$$

Noting that $C_{P_{\max}}$ can be also calculated from the actual power coefficient.

5. Extracted energy by a turbine

The extracted energy from the turbine in a site can be calculated from the following equation;

$$E(v) = f(v) {*} P_e(v) \tag{16}$$

$P_e(v)$ is the power curve of the turbine, and $f(v)$ is the Weibull Distribution. This distribution can be calculated by using the following equation;

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k} \tag{17}$$

where k is the Weibull shape parameter, c is the Weibull scale parameter, and v is the wind speed. The scale parameter can be estimated from the following relation;

$$c = \frac{\overline{V}}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{18}$$