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Technical note

BEM theory: How to take into account the radial flow inside of a 1-D numerical code

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ABSTRACT

Blade Element Momentum (BEM) theory based numerical codes are employed, both in the scientific and industrial field, for designing wind turbines and appraising their performance. Using BEM theory, mono-dimensional mathematical codes can be created. They have very short processing times and high reliability. However, this is related to the solution of a few problems peculiar to these mathematical models.

The problems of these numerical codes are well known in scientific literature: the impossibility of describing inside the mono-dimensional code the three-dimensional radial flow along the blades; the possibility of running into numerical instabilities which prevent the code from converging on the correct solution.

In this work the author will show how to mathematically describe lift coefficients so as to eliminate the lack of description for centrifugal pumping.

Finally, to ascertain the accuracy of numerical results, these will be compared to experimental data taken from scientific literature.

In particular, simulated numerical code data will be compared with the experimental data for the power and efficiency curves of NREL Phase II and NREL Phase VI wind turbines.

1. Introduction

To design a wind turbine, computational codes must be able to evaluate the best wind turbine geometry in order to obtain the best efficiency from the machine and assess its off-design performance. Fluid-dynamic three-dimensional codes (3-D CFD) are widely used. They on one hand produce very accurate results, but on the other need much longer computational times and bigger computational resources. Mono-dimensional codes are far more versatile, but they are little less accurate. Among these, the most applied in universities and industry are those based on the 'Blade Element Momentum (BEM) Theory'.

The Blade Element Momentum Theory was described by Glauert [1]. It is very fast and, provided with reliable airfoil data and considering the refinements described in [2,15,16], yields accurate results.

BEM Theory is based on the actuator disc model which is probably the oldest analytical tool for analysing rotor performance. In this model, the rotor is represented by a permeable disc which allows the flow to pass through the rotor, at the same time being subject to the influence of the surface forces. The classical actuator disc model is based on the conservation of mass, momentum and energy, and constitutes the main ingredient in the 1-D momentum theory, as originally formulated by Rankine [8] and Froude [9]. Combining it with blade element analysis, it is possible to obtain the Blade-Element Momentum Technique [1].

A thorough review of 'classical' actuator disc models for rotors in general and for wind turbines in particular can be found in [10].

BEM Theory based mathematical codes exploit the aerodynamic peculiarities of wing profiles, such as lift and resistance depending on airfoil angle of attack and Reynolds number.

Obviously, these codes are not as reliable as 3-D ones, but offer advantages 3-D codes lack.

For example, BEM Theory based numerical codes do not require great calculation resources, execute calculations in a few seconds on any current Personal Computer, and help carry out all necessary adjustments, in order to optimize the wind rotor geometry, in very short times. A preliminary validation of results from experimental data is indispensable, like for any other numerical code.

This code can assess the rotor geometry which maximizes machine efficiency [21], evaluate turbine fluid-dynamic features [22], as well as off-design working performance (torque, power and related coefficients), maximize energy production [23] and control the wind turbine power curves [24].

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Nomenclature

R	rotor radius [m]
Re	Reynolds number [-]
α	angle of attack [°]
β	angle of attack shift [°]
ϕ	incoming flow direction angle [°]
ω	angular velocity [s^{-1}]
a	axial induction factor [-]
a'	tangential induction factor [-]
r	blade local radius [m]
V_0	wind velocity far up stream [m/s]
N	rotor normal force [N]
c	airfoil chord [m]
ρ	air density [kg/m^3]
C_L	airfoil lift coefficient [-]
C_D	airfoil drag coefficient [-]
C_p	power coefficient [-]
C_q	torque coefficient [-]
C_{Lmax}	C_L at $\alpha = 45°$
C_{Dmax}	maximum C_D value
α_{min}	α value at which starts the airfoil aerodynamic region of attached flow
α_{BC}	α value which splits the dynamic stall region from the fully stalled one.
N_b	number of blades [-]
F	tip loss factor [-]
C_N	normal force coefficient [-]
λ	tip speed ratio [-]
σ	rotor solidity [-]
a_i	C_L logarithmic polynomial coefficients
b_i	C_D logarithmic polynomial coefficients
T	torque [Nm]
P	mechanical power [W]
$P_{Generator}$	electrical power [W]

Abbreviations

BEM	Blade Element Momentum
TSR	Tip Speed Ratio
NREL	National Renewable Energy Laboratory
CFD	Computational Fluid Dynamic
1-D	One-dimensional
2-D	Two-dimensional
3-D	Three-dimensional

2. Mathematical model

The BEM Theory based numerical codes subdivides the wind turbine rotor into annuli of *dr* thickness (Fig. 1), the flow of each sector being independent of adjacent circular sector flows [14]. Applying the equations of momentum and angular momentum conservation, for each infinitesimal *dr* sector of the blade, the axial force and torque can be defined (Eq. (1) and (2)).

The axial force on the blade element of width *dr* is:

$$\mathrm{d}N = \frac{\rho}{2}\frac{V_0^2(1-a)^2}{\sin^2\phi}N_b(C_L\cos\phi + C_D\sin\phi)c\mathrm{d}r \qquad (1)$$

The torque on the blade element of width *dr* is:

$$\mathrm{d}T = \frac{\rho}{2}\frac{V_0(1-a)}{\sin\phi}\cdot\frac{\omega r(1+a')}{\cos\phi}N_b(C_L\sin\phi - C_D\cos\phi)cr\mathrm{d}r \qquad (2)$$

Having knowledge of lift and drag coefficients (C_L and C_D) is of crucial importance for assessing forces and torques according to Eqs. (1) and (2). They are obtained through wind tunnel tests and then mathematically interpolated.

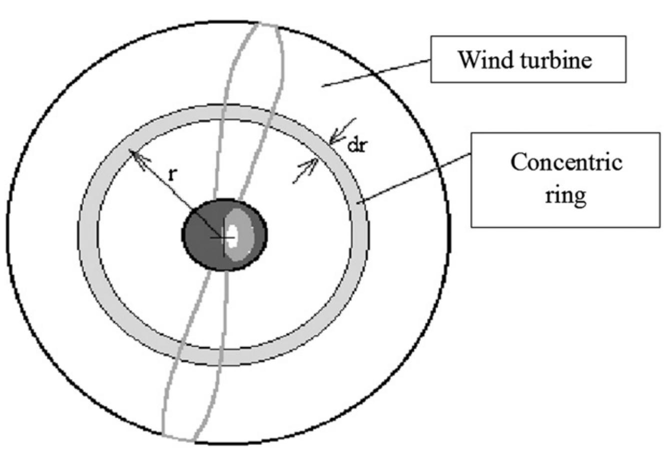

Fig. 1. Wind turbine rotor.

2.1. Mathematical representation of the lift and drag coefficients

A problem which might cause numerical instability is linked to the mathematical description of aerofoil lift coefficient. Unlike aeronautical applications, where airfoils only work under conditions of attacked flow, (stall is an operational condition to be avoided), wind turbines can instead (for reasons of power control, 'stall regulated power') require airfoil to operate under conditions of incipient and/or deep stall.

Aerofoil experimental data are 2-D and taken from wind tunnel measurements. Furthermore, because of rotation the boundary layer is subjected to Coriolis and centrifugal forces which alter the 2-D aerofoil characteristics. This is especially pronounced in stall. It is thus often necessary to extrapolate existing aerofoil data into deep stall and to include the effect of rotation [3–7].

Inside a 1-D numerical code a 3-D effect cannot be included. Only with an mathematical artifice it is possible to take into account the radial flow. In this way, the 1-D numerical code becames enough reliable, and so it is possible to design a wind turbine and evaluate its performance in very short computational times (some second for each simulation vs some day in the 3-D CFD simulation).

Owing to radial flow along the turbine blades, mathematical equations describing lift coefficient have to overestimate experimental C_L values within a precise range of the angle of attack. This is necessary to bypass the impossibility of describing within a 1-D numerical code, the actual three-dimensional effects along the blades of a wind turbine (centrifugal pumping). Centrifugal pumping is a phenomenon which describes radial air flow along blades [11,12]. This flow slows the flow detaching from the airfoil, causing an increase in airfoil lift.

In order to take into account radial flow along a rotating blade, in scientific literature, many authors modified the C_L distribution (see [11,25,26]). All of them incremented the C_L values. Someone incremented the C_L values at the end of the attached flow region (i.e. at the end of the linear trend); others authors incremented the