Renewable Energy 35 (2010) 2468–2475

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Determination of the optimal tilt angle and orientation for solar photovoltaic arrays

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A R T I C L E I N F O

Article history:
Received 10 November 2009
Accepted 6 March 2010
Available online 1 April 2010

Keywords:
Photovoltaic systems
Global solar irradiance
Optimization
Tilt angle
Orientation
Radial basis function (RBF)

A B S T R A C T

This paper deals with the determination of optimum tilt angle and orientation for solar photovoltaic arrays in order to maximize incident solar irradiance exposed on the array, for a specific period of time. The method is extended, by introducing a second objective, i.e. minimization of variance of the produced power, in terms of hourly power generation throughout the given period of time. The proposed method uses both well-established models and data collected from the particular area where the photovoltaic panels will be installed and is built upon four steps. In the first step, the recorded data are used in order to select the most accurate, among several isotropic and anisotropic models that can be found in the literature, for predicting diffuse solar irradiance on inclined surfaces. In the second step, the recorded data and the selected model are used to construct a database that contains the averages and the variances of the hourly global solar irradiance on tilted surfaces over specific periods of time, for various tilt angles and orientations. In the third step, the database of the previous step is utilized to produce meta-models that correlate tilt angle and orientation with mean global irradiance and its variance on tilted surfaces. Finally, an optimization problem is formulated, aiming to determining the optimum values of tilt angle and orientation, taking into account the constraints and limitations of the system.

1. Introduction

The increase of the energy consumption in addition to the need for reducing the emission of pollutants into the atmosphere by using renewable energy sources, has led to a tremendous increase in the use of Photovoltaic (PV) systems [1,2].

A key objective when installing a photovoltaic panel is to achieve the maximum energy output and to avoid shading. To intercept the maximum sunlight, a PV panel must be positioned so that the sun rays arrive at the panel vertically. If not, it does not produce as much power as it could. In order to collect the maximum possible daily energy, one solution is to use tracking systems [3]. A tracker is a mechanical device that follows the direction of the sun on its daily sweep across the sky. However, trackers are expensive and are not always applicable [4]. Thus, in many applications, fixed installations or installations where tilt angle can be adjusted manually are used. It is clear, that in such cases maximum power production can be achieved if the optimal values of tilt and azimuth angles can be determined.

For solar energy applications in the northern hemisphere, optimum orientation is considered to be that of due south. In most cases, PV panels are placed according to this general rule [5,6]. However, there are cases, such as in building-integrated photovoltaic systems (BIPV), where photovoltaic modules are placed in an off-south-facing position, usually according to the façade types and architecture arts [7]. As far as the optimum tilt angle is concerned, this depends on the local latitude. As a rule of thumb, photovoltaics are usually positioned at a tilt angle approximately equal to the latitude of the site and facing south.

Many studies have been performed in order to select the ideal tilt angle of PV panels, based on observation of specific diagrams, empirical relationships and by taking into account detailed characteristics of the site of installation [8–11]. Furthermore, many theoretical models have been suggested by researchers that lead to optimum tilt angles of solar collectors [12–14]. However, the majority of these studies consider the azimuth angle fixed. Obviously, the results can be improved by using two degrees of freedom, i.e. when the design is performed by considering both tilt angle and orientation as free variables.

This paper deals with the determination of both optimum tilt angle and orientation for solar photovoltaic arrays in order to maximize incident solar irradiance exposed on the array, for a specific period of time. The method is extended, by introducing a second

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doi:10.1016/j.renene.2010.03.006

objective, i.e. minimization of variance of the produced power, in terms of hourly power generation throughout the given period of time. This objective is important when we are mostly concerned about satisfying power demands with small variations [15]. The proposed method uses both well-established models and data collected from the particular area, where the photovoltaic panels will be installed and was built upon four steps.

As a first step, the model for predicting global solar irradiance on inclined surfaces was selected. Standard formulas were used for simulating the beam and the reflective components. As far as the diffuse component is concerned, several isotropic and anisotropic models were tested using data that have been collected at the National Technical University of Athens (NTUA) station (37°58′26″B, 23°47′16″A, 219 m above mean sea level). The most accurate model was selected for further study. The result of the first step is a model that can predict hourly global solar irradiance on tilted surface at the specific location where data have been collected, given the tilt angle, the orientation, and the time.

In the second step, the selected model was used to calculate the mean value and the variance of the hourly global solar irradiance on tilted surfaces over a year for a wide range of tilt (0°–90°) and azimuth angles (0° to ±60°). The possibility to change the tilt angle twice a year (summer: April–September and winter: October–March), led to the separate investigation of the hot period and the cold period. A database was thus generated that contains mean values and variances of the hourly global solar irradiance on tilted surfaces for several pairs of tilt angles and azimuth angles.

In the third step mathematical relationships were developed between the design variables (tilt angle and azimuth angle) and output variables (mean value and variance of global irradiance on tilted surfaces), using the database that was generated in the previous step. In particular, the linear regression methodology and the radial basis-function (RBF) neural network architecture were utilized for producing the input–output "meta-models". Several statistical tests were used to validate the produced models, illustrating that the nonlinear approach is the most accurate.

Finally, in order to obtain optimal values for tilt and azimuth angles, a Nonlinear programming (NLP) problem was formulated that takes into account the constraints and limitations of the system.

The methodology offers a new tool to PV system designers, since it incorporates two objectives (maximization of the produced power and minimization of variation in power production), optimizes the design with respect to two parameter, and takes into account the particularities of the system and the climatological and geographical characteristics of the particular location.

2. Recorded data

The data used in this study were recorded at the weather station of the National Technical University of Athens (37°58′26″B, 23°47′16″A, 219 m above mean sea level). These data are values over ten-minute intervals of global, beam, and diffuse irradiance on a horizontal surface, as well as global irradiance on a tilted surface (32°), throughout an one-year period (January 2004 to December 2004) which were then transformed into hourly values, Finally, the available data set contained 8760×4 values and covered a full calendar year. The experimental values for global, diffuse, and beam irradiance on a horizontal surface and the global irradiance on a tilted surface are denoted by I_i, $I_{b,i}$, $I_{d,i}$, $I_{t,i}$ respectively for $i = 1,2,\ldots,8760$.

3. Selection of the model for predicting global solar irradiance on tilted surface(s)

The hourly global solar irradiance incident on inclined surfaces I_t is composed of the beam (I_{tb}), diffuse (I_{td}), and ground-reflected (I_{tr}) components. The beam and ground-reflected components were calculated by using simple models found in the literature [16,17]. The nature of the diffuse component is more complicated and needs more attention.

In particular, the equations for simulating the *beam component* are presented below:

$$I_{tb} = I_b \cdot R_b \tag{1}$$

$$R_b = \cos t_h / \cos t_{hz} \tag{2}$$

$$\cos t_{hz} = \sin \varphi \cdot \sin \delta \cdot \cos \varphi \cdot \cos \delta \cdot \cos\omega \tag{3}$$

$$\begin{aligned} \cos t_h = {} & \sin \delta \cdot \sin \varphi \cdot \cos b - \sin \delta \cdot \cos \varphi \cdot \sin b \cdot \cos g \\ & + \cos \delta \cdot \cos \varphi \cdot \cos b \cdot \cos \omega \\ & + \cos \delta \cdot \sin \varphi \cdot \sin b \cdot \cos g \cdot \cos \omega \\ & + \cos \delta \cdot \sin b \cdot \sin g \cdot \sin \omega \end{aligned} \tag{4}$$

$$\delta = 23.45 \cdot \sin[2\pi \cdot (284 + n)/365] \tag{5}$$

where I_{tb} is the hourly beam irradiance on inclined surfaces, t_{hz} is the solar zenith angle, t_h is the solar incidence angle on a tilted plane, φ is the latitude of the location, δ is the solar declination, b is the tilt angle, ω is the solar hour angle, g is the azimuthal angle and n is the day of the year starting from the 1st of January.

The solar hour angle ω is defined as the angular displacement of the sun on the apparent orbit (ecliptic) east or west of the local meridian (it is zero at solar noon and varies by 15° per hour from solar noon, it is negative in the morning and positive in the afternoon). The azimuth angles for the northern hemisphere, range between −90° and 90° (i.e. $g = 0°$ for south-facing, $g = -90°$ for east-facing, $g = 90°$ for west-facing surfaces).

The *ground-reflected* component was computed by the following formula

$$I_{tr} = 1/2 \cdot r_g \cdot I \cdot (1 - \cos b) \tag{6}$$

where r_g is the ground reflectivity and I is the global solar irradiance on horizontal surfaces.

It must be noted that the aforementioned method presumes that the components of global solar irradiance on the horizontal surface are known. If not, the appropriate decomposition model should be selected for their calculation [18]. Furthermore, in remote areas where data of global solar irradiance is sparse or unavailable, many studies found in the literature can provide accurate predictions [19,20].

As far as the diffuse component is concerned, several empirical methods can be found in the literature, which can be classified as isotropic and anisotropic. The isotropic models assume the uniformity of diffuse sky irradiance over the sky dome. The anisotropic models assume the anisotropy of the diffuse sky irradiance in the circumsolar region (sky near the solar disk) and an isotropically diffuse component for the rest of the sky dome. The diffuse models that appear in Table 1 were tested on the collected data (see Section 2) as follows: For each one of the 8760 recorded values an estimate of the hourly global irradiance on a tilted surface of 32° was produced using the following equation:

$$\widehat{I}_{t,i} = I_{tb,i} + I_{tr,i} + \widehat{I}_{td,i} \tag{7}$$

where $I_{tb,i}$, $I_{tr,i}$ were calculated by Eqs. (1)–(5) and (6) respectively for $i = 1,2,\ldots,8760$, while $\widehat{I}_{td,i}$ is the prediction of the diffuse model that was tested. The predictions obtained by Eq. (7) were tested against the experimental $I_{t,i}$ values for each possible choice for