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Technical Note

Horizontal axis wind turbine working at maximum power coefficient continuously

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ABSTRACT

The performance of a horizontal axis wind turbine continuously operating at its maximum power coefficient was evaluated by a calculation code based on Blade Element Momentum (BEM) theory. It was then evaluated for performance and Annual Energy Production (AEP) at a constant standard rotational velocity as well as at a variable velocity but at its maximum power coefficient.

The mathematical code produced a power coefficiency curve which showed that notwithstanding further increases in rotational velocity a constant maximum power value was reached even as wind velocity increased.

This means that as wind velocity varies there will always be a rotational velocity of the turbine which maximises its coefficient. It would be sufficient therefore to formulate the law governing the variation in rotational velocity as it varied with wind velocity to arrive at a power coefficient that is always the same and its maximum.

This work demonstrates the methodology for determining the law governing the rotational velocity of the rotor and it highlights the advantages of a wind turbine whose power coefficient is always at maximum rather than very variable in line with the variation of wind velocity.

1. Introduction

Design parameter choice is critical for optimising wind turbine performance. For any fixed diameter there are various parameters influencing energy production: rotor rotation velocity, blade number, airfoil chord distribution and longitudinal blade twist.

In this paper the influence of rotor rotational velocity on wind turbine performance has been investigated. In particular, it has been observed that there exists a mathematical law on the variation of rotor rotational velocity with wind speed, which allows the wind turbine to always operate at its maximum power coefficient.

It is well known that a wind turbine power coefficient presents a maximum value for a particular wind speed, which decreases rapidly for all other wind velocities. Vice versa, varying the rotor rotational velocity at different wind speeds, it is possible to have a power coefficient which is always at its maximum value.

To investigate this analysis thoroughly, and to evaluate the mathematical law on rotor rotational velocity, the authors applied a numerical model [3], based on BEM theory [1,2], and validate it through experimental measurements [4].

The mathematical model based on BEM (Blade Element Momentum) theory is the most frequently used by Science and Industry [4,6–12]. It enables the design of rotor blades by fluid dynamics, and the evaluation of wind turbine performance (in design and off-design conditions). Using this model, it is possible to design the rotor, choose the geometric characteristics of the turbine (rotor diameter, aerodynamic airfoils, chord, pitch and twist), and evaluate the forces acting on the blades, the torque and power at the rotor shaft. It is also possible to evaluate turbine performance with a wide range of wind velocities.

The BEM theory is based on the Glauert propeller theory [12], modified for application to wind turbines. In recent years the BEM theory has been optimized and modified to provide increasingly accurate results. For the numerical stability of the mathematical model the greatest difficulties are determining axial and tangential induction factors, the lack of experimental measurements on airfoil lift and drag coefficients at high angles of attack, and their three-dimensional representation. In order to take the three-dimensional representation into account, the wind tunnel experimental measurements must be modified in order to consider radial flow along the blades (centrifugal pumping [4]).

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Nomenclature

a	axial induction factor
a'	tangential induction factor
r_1	blade local radius
c	airfoil chord
c_i	twist logarithmic polynomial coefficients
a_i	C_L logarithmic polynomial coefficients
b_i	C_D logarithmic polynomial coefficients
c'	Weibull scale parameter
n	rotor rotational velocity
w_s	wind speed
v	wind velocity
$\bar{v}$	mean wind velocity
WT	wind turbine
AEP	annual energy production
BEM	blade element momentum
HAWT	horizontal axis wind turbine
R_1	wind rotor radius
Re	Reynolds number
V_0	wind velocity far up stream
N	rotor normal force
C_L	lift coefficient
C_D	drag coefficient
N_b	number of blades
M	torque
F	Tip loss factor
C_P	power coefficient
C_N	normal force coefficient
P	power
K	Weibull shape parameter
P_w	power of a wind machine
E_w	energy from a wind machine
θ	twist
α	angle of attack
φ	incoming flow direction angle
ρ	air density
λ_r	local speed ratio

2. Mathematical code

The mathematical model for the fluid dynamics wind turbine design (and for the WT performance evaluation), developed in a previous work [3], is based on Blade Element Momentum Theory. By applying momentum and angular momentum conservation equations, the axial force and torque acting on the blade sector is obtained (as given in Equations (1) and (2)),

$$dN = \frac{\rho}{2}\frac{V_0^2(1-a)^2}{\sin^2\phi} N_b\,(C_L\cos\phi + C_D\sin\phi)\,c\,dr_1 \tag{1}$$

$$dM = \frac{\rho}{2}\frac{V_0(1-a)}{\sin\phi}\cdot\frac{\omega\,r_1(1+a')}{\cos\phi} N_b\,(C_L\sin\phi - C_D\cos\phi)\,c\,r_1 dr_1 \tag{2}$$

and thus the torque M at the rotor shaft is the summation of dM for all the blade sectors.

The wind turbine power is given by $P = M^*\omega$, where ω is the rotor angular velocity.

The lift (C_L) and drag (C_D) coefficients for a given airfoil are evaluated from wind tunnel measurements [6]. The experimental values were fitted to obtain mathematical functions to apply to the simulation model. To fit the experimental data to the lift and drag coefficients, a fifth-order logarithmic polynomial (shifted by 10 degrees) was implemented for the 'Attached Flow Regime' and 'High Lift, Stall Development Regime – Dynamic Stall' (see [3] for the aerodynamic regions definition), as shown in Equations (3) and (4).

As described in [3], centrifugal pumping (3D aerodynamic effects) was taken into account (Equation (5)) with a slight increment in the C_L experimental values in the early region of the "Flat Plate, Fully Stalled Regime".

$$C_L = \sum_{i=0}^{5} a_i[\ln(\alpha+10)]^{\,i} \tag{3}$$

$$C_D = \sum_{i=0}^{5} b_i[\ln(\alpha+10)]^{\,i} \tag{4}$$

The a_i and b_i coefficients were determined by means of the least square method, fitting experimental data for the S809 airfoil at $Re = 10^6$.

For the 'Flat Plate, Fully Stalled Regime' the mathematical functions of Equations (5) and (6) were implemented.

$$C_L = 2\,C_{L_{max}}\cdot\sin\alpha\cdot\cos\alpha \tag{5}$$

$$C_D = C_{D_{max}}\cdot\sin^2\alpha \tag{6}$$

In Equation (7) $C_{L_{max}}$ and $C_{D_{max}}$ are shown.

$$C_{L_{max}} = C_L|_{\alpha=45^\circ} \text{ and } C_{D_{max}} = C_D|_{\alpha=90^\circ} \tag{7}$$

The numerical stability of the mathematical code depends on tangential and axial induction factors. In this code the induction factors, reported in Equations (8)–(10), were implemented [3].

For $a < 0.4$:

$$a = \frac{1}{\dfrac{4F\sin^2\phi}{\frac{c\,N_b}{2\pi\,r_1}(C_L\cos\phi + C_D\sin\phi)} + 1} \tag{8}$$

while for $a \geq 0.4$ [7]:

$$a = \frac{18F - 20 - 3\sqrt{C_N(50-36F) + 12F\,(3F-4)}}{36F - 50} \tag{9}$$

and

$$a' = \frac{1}{2}\left(\sqrt{1 + \frac{4}{\lambda_r^2}a(1-a)} - 1\right) \tag{10}$$

As reported in [4] and [8], F is the Prandtl Tip Loss Factor, and is defined as:

$$F = \frac{2}{\pi}ar\cos\left[\exp\left(\frac{N_b\,(r_1 - R_1)}{2\,r_1\sin\phi}\right)\right] \tag{11}$$

To verify the validity of the mathematical code, the simulated data was compared with the NREL data from the NASA-Ames wind tunnel tests [4]. The UAE Phase VI wind turbine has two twisted blades, a variable chord along the blade, and a rotor diameter of 10 m [5]. The aerodynamic airfoil is the S809 and is constant along the blades; the pitch is three degrees and rotational velocity is 72 r/min. Fig. 1 shows a comparison between the simulated and