Renewable Energy 34 (2009) 2252–2258

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Technical Note

A fast algorithm for the hourly simulations of ground-source heat pumps using arbitrary response factors

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ARTICLE INFO

Article history:
Received 13 June 2007
Accepted 3 February 2009
Available online 6 March 2009

Keywords:
Ground source heat pump
Ground heat exchanger
Inverse Laplace transforms

ABSTRACT

Hourly energy simulations are an important part of the design and analysis of ground-source heat pump systems. In order to evaluate the fluid temperature in the borehole of a geothermal heat pump system, most of the current models express the heat transfer rate as a sum of step changes in heat transfer rate. The borehole temperature is then computed as a superposition of the different contributions of each time step. The main difference between the different models lies in the way the step response is computed. Since all these methods are based on a convolution scheme, long time simulations are very time consuming. Many load aggregation algorithms have been proposed in order to reduce this computational time. In a previous paper we proposed a new algorithm to evaluate the overall response which was much faster than the classical aggregation schemes. However this new algorithm was based on the cylindrical source step response for a single borehole. In this paper, we present a generalization of this scheme for any kind of step response making it a very powerful tool for hourly simulations.

1. Introduction

The modeling of ground-source heat pump systems (GSHPs) is a very active research field in recent years. The main goal is to have a good design tool for bore field design since it is a very expensive part of a GSHP system and also to have a good tool to analyse the economical benefit of such an expensive system.

The usual design approach for GSHP systems is to evaluate peak monthly and daily loads and some average annual loads and to evaluate a minimum length to achieve enough heat transfer for the system to work [1]. In recent years, the simulation of these systems in response to hourly loads are becoming more and more popular in order to predict their performance more precisely [2–4]. Hourly simulations for long period of time are also an important tool to evaluate the economical benefit of GSHP systems over the years. Simulations over 10, 20 and even 30 years are often looked for.

Simulation software like TRNSYS are available to achieve such simulations. Type 557 based on the duct storage model (DST) of Hellström [5] serves often as a reference and is often used to compare new approaches to ground heat exchanger simulations. It is surprisingly fast and can simulate long period of time. It has however inherent simplification hypothesis and one need to have the simulation software to use it. Other approaches to treat the problem is to evaluate the thermal response of geothermal boreholes based on the superposition of heat steps at certain period of time, often chosen as 1 h. Such schemes are basically convolution schemes and can be very time consuming if they are treated as such. The differences in the models are mostly in the way the heat conduction problem in the ground is solved, the way the interference problem between boreholes is treated and the way to accelerate the schemes. A good survey of the different models is given by Sheriff [6]. We may split these methods in two main approaches: analytical and numerical methods. Some well known analytical solutions are the line source model [7], the cylindrical heat source model [8]; both are solutions of the unsteady radial problem T(r, t). Some solutions treat the axial problem [9,10]. Other solutions based on numerical solutions were proposed by Eskilson [11], Yavuzturk and Spitler [12,13] and Xu and Spitler [4]. The last ones being essentially extrapolations of the first one to take into account the short-time behaviour of the thermal response.

Almost all accelerating methods proposed are based on aggregation schemes. Basically in the classical convolution scheme, the weights associated to all hourly loads are recalculated at each time. In an aggregation scheme, "old loads" are aggregated to form a mean load over a period of time. The differences depend essentially on the aggregated period chosen: month, year or days, etc. Bernier et al. [3], Yavuzturk and Spitler [12,13] propose different schemes to the problem. The present author [14] proposed a new approach which was much faster than the aggregated schemes. The method will be revised in the next section. However, the method

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doi:10.1016/j.renene.2009.02.010

Nomenclature

F_n	coefficients, defined in (11)
g	g-function
G	G-function
H	Borehole height (m)
J	Bessel function of the first kind
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
$\mathbf{L}$	Laplace transform
q	heat load (W)
q'	heat flux per unit length ($W\,m^{-1}$)
r	radial coordinate (m)
R'	unit length thermal resistance ($K\,m\,W^{-1}$)
T	temperature (K)
u	unknown function, defined in (22)
v	function, defined in (10)
Y	Bessel function of the second kind
z	dummy integration variable

Greek letters

α	Thermal diffusivity ($m^2\,h^{-1}$)
γ	$(3r_b/H)^2$
τ	time integration variable

Subscripts

b	at the borehole radius
f	fluid
g	ground values
o	far-field value

was based on the cylindrical heat source step response due to its particular mathematical form. In the present paper we propose the extension of this scheme to any kind of step response.

2. Solution with the cylindrical heat source

Several approaches can be taken to evaluate the thermal performance of vertical geothermal heat exchanger (Fig. 1). The simplest is to define a mean fluid temperature $T_f(t)$and a mean borehole temperature $T_b(t)$

$$T_f(t) - T_b(t) = q_b'(t)R_b' \tag{1}$$

$$T_b(t) - T_o = q_b'(t)R_g'(t) \tag{2}$$

If we neglect axial temperature variation, the solution of problem (2) is to find the temperature distribution $T(r, t)$ satisfying the heat conduction equation,

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{3}$$

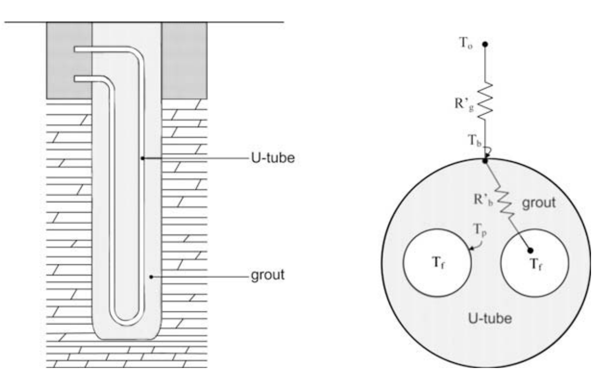

Fig. 1. Schematic of the borehole.

for the domain $r > r_b$, $t > 0$, and the following boundary conditions

$$T(r,0) = T_o \quad , \quad -k\frac{\partial T}{\partial r}\bigg|_{r=r_b} = q_b''(t) = \frac{q_b'(t)}{2\pi r_b} \tag{4}$$

where q_b' is the heat flow per unit length (q_b' is often referred as the heat entering the borehole. Here, we keep the normal convention as the heat in the positive radial direction). In the case of step-function $q_b'(t) = q_b'\,u(t)$, the solution is well known and it is given in the classical book of Carslaw and Jaeger's [8]

$$T(\tilde r,\tilde t) - T_o = \frac{q_b'}{k}\underbrace{\frac{1}{\pi^2}\int_0^\infty \frac{e^{-z^2\tilde t}-1}{z^2\left(J_1^2(z)+Y_1^2(z)\right)}\left[J_0(\tilde r z)Y_1(z) - J_1(z)Y_0(\tilde r z)\right]dz}_{G(\tilde r,\tilde t)} \tag{5}$$

where $\tilde r = r/r_b, \tilde t = \alpha t/r_b^2$ = Fourier number.

The solution of (2) is found by evaluating this last solution at $r = r_b$

$$T(1,\tilde t) - T_o = T_b(\tilde t) - T_o = \frac{q_b'}{k}\underbrace{\frac{2}{\pi^3}\int_0^\infty \frac{1-e^{-z^2\tilde t}}{z^3\left(J_1^2(z)+Y_1^2(z)\right)}dz}_{G(1,\tilde t)} \tag{6}$$

Since

$$J_0(z)Y_1(z) - J_1(z)Y_0(z) = \frac{-2}{\pi z} \tag{7}$$

This solution is known as the cylindrical heat source method (CHSM) also called *G-function* not to be confused with the Eskilson's *g-function* described in the next section. Workers using this solution for the analysis of GCHP systems [15,16], use an analytical approximation of the *G-function* in their computation with the extension of arbitrary loads with the following expression:

$$T(\tilde r,\tilde t) - T_o = \frac{1}{k}\sum_{i=1}^{N}(q_i - q_{i-1})G\left(\tilde r, \frac{\alpha(t-t_{i-1})}{r_b^2}\right) \tag{8}$$

The present author [14] proposed a much faster solution for this particular case than aggregation schemes. The exponential time