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Time-domain stochastic finite element simulation of uncertain seismic wave propagation through uncertain heterogeneous solids

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ABSTRACT

This paper presents an efficient numerical methodology in probabilistically solving the governing partial differential equation of solid mechanics with uncertainties in both the material parameter and forcing function in the time domain using the stochastic Galerkin approach. The methodology hypothesizes the input forcing function and the elastic modulus of the solid to be a nonstationary random process and a heterogeneous random field, respectively, and efficiently represents them in terms of multidimensional Hermite polynomial chaos – orthogonal and uncorrelated polynomials of zero-mean, unit variance Gaussian random variables – by taking advantage of the optimality of the Kosambi-Karhunen-Loève theorem. The methodology allows for any non-Gaussian marginal distributions and any arbitrary correlation structures for the input process and field. The solution random processes (displacement, velocity, and acceleration) are also represented in terms of multidimensional Hermite polynomial chaos expansions whose coefficients at each time step are estimated by applying a stochastic Galerkin projection with the time integration performed via the Newmark's method. The methodology is illustrated, keeping the geotechnical site response analysis in mind, with fully probabilistic, time-domain propagations of bedrock motions through an elastic soil deposit in one-dimension, and is verified using the Monte Carlo method. The effects of input uncertainty parameters of the soil modulus and bedrock motion on the simulated surface motion are also quantified through a parametric sensitivity study.

1. Introduction

Presence of inevitable uncertainties and the need to account for them explicitly in our predictions have long been recognized by the earthquake engineering community. Pioneering works by late Professor C. Allin Cornell during late 1960s [11] and push by the Pacific Earthquake Engineering Research Center (PEER) during early 2000s [12,47] saw the development of performance-based design framework to account for those uncertainties in our design philosophy. However, numerical simulations of the behavior of solids and structures under seismic loading – which are increasingly being used to feed the performance-based design framework – still remain largely deterministic, amid the presence of huge uncertainties in the systems. This is mainly due to the issue of computational tractability of the Monte Carlo approach [46] in solving the governing partial differential equation (PDE) of solid mechanics with uncertain operators/coefficients and uncertain forcing function.

Among alternate approaches for solving uncertain PDEs, there exists analytical technique where the only uncertain parameters are in the external forcing functions. For such a PDE, the probability density function (PDF) of the solution variable satisfies a Fokker-Planck-Kolmogorov (FPK) equation [33,56]. Mathematical tools, however, are not that well developed for PDEs with operators/coefficients uncertainty. Exact solution to problems with stochastic operators was attempted by Hopf [28], using the characteristic functional approach. Later, Lee [38] applied the methodology to the problem of wave propagation in random medium and derived an FPK equation, satisfied by the characteristic functional of the random wave field. This characteristic functional approach, though elegant, is not completely rigorous and very difficult to extend in solving realistic problems with irregular geometries and boundaries. The difficulty in analytical solution led to development of approximate techniques. Among the early approximate approaches, the perturbation technique [9] is the most common. It is, however, limited to systems with small uncertainties in model parameters. It also suffers from the closure requirement – higher order statistical moments are needed to compute the lower order statistical moments. Frisch [19] provided a very thorough mathematical review of the early analytical and approximate methods in relation to the theory of (mostly acoustic

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and electromagnetic) wave propagation in random media. Early works related to seismic wave propagation through random geologic media also have mostly relied on the perturbation technique [59,42,52,64]. A review of early developments in the field of stochastic soil dynamics was compiled by Manolis [41].

In recent years, with the availability of faster computers, large scale probabilistic simulations are being attempted in other fields of science and engineering using advanced numerical approaches. Among these approaches, the stochastic collocation and the stochastic Galerkin approaches are more common. Stochastic collocation approaches [4,43,61,63,7] can be viewed as a Monte Carlo type sampling technique, with the exception that, instead of at random, the sampling points are selected following some kind of numerical quadrature schemes which are used to estimate the statistical moments of the solution variable. These approaches are non-intrusive approaches as they do not require any modifications to the underlying deterministic code; the stochastic collocation scheme just acts as a wrapper on the deterministic code. Stochastic Galerkin approaches [14,24,44,60,62,7], on the other hand, are, in general, intrusive approaches in the sense that they require modifications to the deterministic code; see Refs. [1,27] regarding non-intrusive stochastic Galerkin approaches. Intrusive stochastic Galerkin approaches usually represent the unknown solution variable using some type of finite series expansions (mostly spectral, e.g., polynomial chaos (PC) expansion [58]) and then employ a Galerkin technique to minimize the errors of finite representation which result in a system of coupled equations. While there is no unanimous agreement in the research community on which approach is better for solving a given problem, it is generally accepted that stochastic Galerkin approaches are more efficient as their accuracies are optimal [60,16,6,27]. In this context, it is important to mention that, depending upon the size of the problem, the stochastic Galerkin method may require inversion of an extremely large matrix. However, the matrix is usually very sparse and has special block properties which are typically exploited for a faster solution [22,26]. Moreover, some studies [40,45] also have demonstrated that solution of such large coupled system of equations may also be advantageous over independently solving many smaller systems of equations (as in the case of the stochastic collocation method) since information on convergence behavior from one block to another can be transmitted during the solution process, resulting in speed up of computation.

In the field of solid mechanics, stochastic Galerkin approaches are so far employed mainly to solve static problems – both linear (elastic) and nonlinear (elastic-plastic as well as geometric) – with uncertain material parameters [21,23,2,26,55,3,31]. Solutions of dynamic problems are also attempted, but mostly in the frequency domain [24,25], thereby restricting the use of the algorithms only to linear problems. Kundu and Adhikari [36], very recently, have presented a time domain formulation of a stochastic Galerkin scheme for application in the field of structural dynamics. They, however, have considered uncertainty only in the material parameter – hypothesizing it to be a Gaussian random field while assuming the forcing to be a deterministic function.

We present a time-domain stochastic finite element formulation, based on an intrusive stochastic Galerkin approach, for solving the governing PDE of solid mechanics with uncertainty in both material parameter(s) and forcing function. Moreover, our formulation allows for any arbitrary non-Gaussian marginal distributions and any arbitrary heterogeneous/nonstationary correlation structures for the material parameter random field and forcing random process. Conventional approaches for discretizations of input random fields and processes within a stochastic Galerkin scheme typically rely on Kosambi-Karhunen-Loève (KKL) expansion [34,32,39] to represent the input random field/process into an optimal number of orthogonal random variables. For Gaussian random fields/processes, the resulting random variables are also uncorrelated (independent), which is a very desirable property as it significantly reduces the overall computational effort. For non-Gaussian random fields/processes, however, the KKL expansion yields correlated random variables which need special treatment, resulting in additional computational burden. Our formulation uses a combination of PC and KKL expansions [54] to efficiently discretize any arbitrary non-Gaussian input random fields/processes directly into a finite number of orthogonal and independent random variables, thereby simplifying the solution process.

The salient features of the formulation are highlighted through two example simulations which are designed keeping the geotechnical site response analysis in mind. Both examples involve one-dimensional propagation of bedrock motion (shear wave) through an elastic soil deposit. While the first example assumes the only uncertainty to be in the soil parameter (shear modulus), the second example considers uncertainties in both the soil parameter and bedrock motion. The simulation results are presented in terms of the marginal mean, marginal standard deviation, and marginal PDF of the surface motion time histories. A parametric study is also performed to quantify the effect of the input uncertainty parameters of the soil modulus and bedrock motion on the simulated surface motion.

2. Formulation of dynamic time-domain stochastic finite element method

The governing equations for a three-dimensional elastic solid in the Cartesian coordinate system are:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = \rho \ddot{u}_i \quad \text{(equilibrium equation)} \tag{1}$$

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad \text{(compatibility equation)} \tag{2}$$

$$\sigma_{ij} = D_{ijkl}\epsilon_{kl} \quad \text{(constitutive equation)} \tag{3}$$

where σ, ϵ, D, ρ, b, u, and $\ddot{u}$ are the stress, strain, material constitutive parameter, material density, body force, displacement, and acceleration, respectively. Neglecting the body force and employing the Galerkin weak formulation of deterministic, linear, dynamic finite elements [30], the above equations can be written as:

$$\sum_e \left[\int_{D_e} N_m(x)\rho(x)N_n(x)d\Omega \ddot{u}_n(t) + \int_{D_e} \nabla N_m(x)D(x)\nabla N_n(x)d\Omega u_n(t) - f_m(t) \right] = 0 \tag{4}$$

where N_m is the finite element shape function, while $\sum_e$ denotes the assembly procedure over all finite elements of the discretized domain, Ω and $f_m(t)$ incorporates the various elemental contributions to the global force vector.

We will next assume the material constitutive parameter, $D(x)$, and the forcing function, $f_m(t)$, to be a heterogeneous random field and a non-stationary random process, respectively and represent them in terms of multidimensional, Hermite PC expansions with known coefficients. As a result, the nodal displacement, $u_n(t)$, and nodal acceleration, $\ddot{u}_n(t)$, will also become random processes. They will also be represented using multidimensional, Hermite PC expansions but with unknown coefficients which will be computed using a stochastic Galerkin approach.