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Wavelet-based generation of spatially correlated accelerograms

Kaushik Sarkar, Vinay K. Gupta*, Riya C. George

Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India

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ABSTRACT

For the seismic analysis of complex or nonlinear extended structures, it is useful to generate a set of properly correlated earthquake accelerograms that are consistent with a specified seismic hazard. A new simulation approach is presented in this paper for the generation of ensembles of spatially correlated accelerograms such that the simulated motions are consistent with (i) a parent accelerogram in the sense of temporal variations in frequency content, (ii) a design spectrum in the mean sense, and (iii) with a given instantaneous coherency structure. The formulation is based on the extension of stochastic decomposition technique to wavelet domain via the method of spectral factorization. A complex variant of the modified Littlewood-Paley wavelet function is proposed for the wavelet-based representation of earthquake accelerograms, such that this explicitly brings out the phase information of the signal, besides being able to decompose it into component time-histories having energy in non-overlapping frequency bands. The proposed approach is illustrated by generating ensembles of accelerograms at four stations.

1. Introduction

The primary concern in the analysis and design of structures for the effects of strong earthquakes is the proper definition and representation of the design ground motion. Among different approaches used to this end, those via response spectrum or power spectral density function (PSDF) are most common. However, in several applications, such as performance evaluation of mathematical models of structures for design level motions, experimental verification of new design concepts, and statistical analyses of complex and nonlinear structures, it is required to have a time-description of the desired ground motion. In most cases, the available recorded ground motions may not meet the necessary design specifications for a given site. Therefore, there remains a need for the simulation of artificial ground motions compatible with the design requirements.

For the analyses of spatially extended structures, such as long-span bridges, pipelines or even a simple building system with raft foundation, it is required to account for possible variations in the earthquake ground motion at different points in space. Spatial variability in seismic ground motions can result from a number of causes, such as, wave passage effect, incoherence effect, extended source effect, attenuation effect etc. The spatial variability of ground motions has been estimated and modeled stochastically by using the strong motions recorded at dense instrument arrays. Ground motions at different stations are typically considered to be the realizations of space-time random fields. Spatial variability is characterized by the coherency function, which is defined for any two homogeneous random processes in terms of their smoothed cross-PSDF and individual PSDFs. Based on the regression analyses of the data available from the dense instrument arrays (SMART-1 array, LSST array, etc.), a number of empirical and semi-empirical models have been proposed for the coherency function [1–13].

Generation of spatially correlated accelerograms has been attempted by several researchers. The techniques used for this purpose include spectral factorization [7,14–17], covariance matrix decomposition [18], auto-regressive moving average (ARMA) approximation [19], sinusoid superposition [20,21], fast Fourier transform and digital filtering-based methods [22,23], and conditional simulation [24–27]. The main objective in these simulation schemes was that the statistical properties of the simulated motions matched with those of the target random field. Some of these schemes have used the method of stochastic decomposition suggested by Shinozuka [28]. Hao et al. [7] generated a set of correlated time histories by using the summation of trigonometric series. Li and Kareem [22] used time-dependent weighing functions in the stochastic decomposition, while the target ground motion characteristics were specified in terms of an evolutionary spectral matrix. Shrikhande and Gupta [17] generated spatially correlated time histories by using the nonstationary characteristics of a given accelerogram. Zerva [29] has reviewed various schemes

* Corresponding author.
E-mail address: vinaykg@iitk.ac.in (V.K. Gupta).

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of simulation of ground motions in detail.

In most of the above simulation procedures, except for those proposed by Li and Kareem [22] and Shrikhande and Gupta [17], generated time histories were modulated with the help of a deterministic envelope function. However, the envelope function and phase spectrum of a time-history are known to be closely related [30,31], and therefore, such modulation may change the phase properties arbitrarily, thus disturbing the coherency structure. Also, it is unrealistic to model a complex phenomenon like earthquake ground motion via a deterministic modulating function. The scheme proposed by Shrikhande and Gupta [17] incorporates nonstationarity in the simulation procedure itself by using the phase and duration spectra of a recorded time-history, thus requiring no post-processing of the simulated motions and keeping the coherency structure intact. However, this scheme does not account for temporal variations in the coherency structure. Also, the energy distributions in the target spectrum and the parent accelerogram need to be "not too different".

There have been several attempts in the recent years that have focused on simulating more realistic spatially correlated accelerograms. Bi and Hao [32] approximately simulated the spatially varying ground motions at an uneven site with nonuniform soil conditions. Konakli and Der Kiureghian [33] simulated nonstationary ground motions considering the effects of incoherence, wave passage and differential site response. Cacciola and Deodatis [34] illustrated the simulation of ground motions at stations with different soil conditions and separated by 30–50 m of distance. Zhang et al. [35] simulated tri-directional nonstationary accelerograms at varying site conditions by considering power spectra at the bed rock and the site amplification of P-, SV- and SH-waves. In a more recent publication, Shields [36] simulated spectrum-compatible, uniformly modulated nonstationary accelerograms by upgrading the evolutionary power spectral density function with random pulse-like perturbations.

Considering that nonstationarity is directly linked to temporal variations in the characteristics of a signal, a time-frequency transformation tool is needed to simulate realistic accelerograms. For example, Wen and Gu [37] simulated nonstationary processes based on Hilbert spectra. The development of wavelet transform technique has however made it possible to represent the temporal variations in the frequency content of a signal more elegantly. The wavelet transform technique is more versatile than the other time-frequency localizing techniques, like Gabor transform, short-time Fourier transform, etc., due to its flexible time-frequency windowing feature [38]. Besides several important engineering applications [39–45], this technique has already been used by [46,47] for the characterization of design ground motions. Zeldin and Spanos [48] synthesized random fields using wavelets. Spanos and Failla [49] and Huang and Chen [50] estimated the evolutionary spectra using wavelet transforms. Iyama and Kuwamura [51] and Gurley and Kareem [52] simulated ground motions using wavelet transforms. Cecini and Palmeri [53] and Giaralis and Spanos [54] simulated spectrum-compatible accelerograms using harmonic wavelets. Huang [55] simulated nonlinear spatially variable ground motions using wavelets and spectral representation method.

In this study based on the thesis of the first author [56], a wavelet-based procedure is formulated for simulating the ensembles of spatially correlated accelerograms, such that those are compatible with a given response spectrum and an assumed coherency model. An analytic function is considered as the mother wavelet function and the popular stochastic decomposition technique is extended to the wavelet domain for this purpose. The proposed approach is illustrated by generating a set of ensembles of correlated accelerograms for the stations 100, 200 and 300 m apart.

2. Wavelet transform

2.1. Brief review

If $f(t)$ is a function belonging to $L^2(R)$ space, the continuous wavelet transformation of $f(t)$ with respect to a mother wavelet function $\psi(t)$ is defined as [38,57]

$$W_\psi f(a,b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}^*(t)\mathrm{d}t \tag{1}$$

with

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \tag{2}$$

where a and b are real-valued scale and shift parameters, respectively, and the asterisk denotes complex conjugation. The transient nature and finite energy content of the earthquake signals make it possible to have their wavelet domain representation. It is possible to reconstruct the original signal $f(t)$ from its wavelet coefficients $W_\psi f(a,b)$ as

$$f(t) = \frac{1}{2\pi C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{a^2}W_\psi f(a,b)\psi_{a,b}(t)\mathrm{d}a\mathrm{d}b \tag{3}$$

with

$$C_\psi = \int_{-\infty}^{\infty}\frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|}\mathrm{d}\omega \tag{4}$$

and

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(t)e^{-i\omega t}\mathrm{d}t \tag{5}$$

denoting the Fourier transform of $\psi(t)$.

2.2. Mother wavelet

The choice of mother wavelet depends on the type of application and the type of function being analyzed. The most common transformation technique, that is Fourier transformation, uses a complex basis function, $e^{i\omega t}$. This function consists of a real (cosine) function added in quadrature to its Hilbert transform (i.e., sine function). Due to this, $e^{i\omega t}$ belongs to a special class of complex functions, called analytic functions, which have non-zero spectra only for positive frequencies [58]. The use of an analytic function as the basis function entails it to reveal the phase information of a signal, and therefore, Fourier transform is considered to be useful for deriving the phase properties of stationary signals. Unlike the Fourier transformation, a wavelet transformation uses a time-localized oscillatory function as the analyzing or mother wavelet, which can be either real or complex. Both real and complex mother wavelets perform a complete and reversible transformation of a signal from time domain to wavelet domain with no information loss, but in the case of real wavelets, the phase-related information of the signal cannot be separated out from the transformed signal. It is therefore necessary that whenever instantaneous phase properties of a signal are explicitly required, a complex mother wavelet, which is also an analytic function, is used [58].

Another important characteristic of the mother wavelet function is its resolution. In this respect, the mother wavelet proposed by Basu and Gupta [41] is well suited to deal with earthquake signals. This function is basically a modified version of the Littlewood-Paley (L-P) wavelet function, with improved resolution in frequency domain. The advantage of the L-P basis function is that its Fourier spectrum is constant over a specific band of frequencies and zero for all other frequencies. However, with the original