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Analysis of wave propagation and soil–structure interaction using a perfectly matched layer model

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ABSTRACT

This paper presents an application of a perfectly matched layer as an absorbing condition for solution of linear wave equations for unbounded domains. The perfectly matched layer can be combined with different numerical methods. In this particular case, a frequency dependent finite element formulation has been pursued. What is most significant is that it utilizes a newly established requirement for definition of the perfectly matched layer, which improves the accuracy and efficiency of the method. In this study, different definitions of the attenuation were considered to optimize the performance of the radiation condition. They were verified in comparison with an exact solution of wave propagation in a half-plane. This analysis was used later to establish a requirement for definition of minimum layer depth as an accuracy prerequisite. Similar requirements are presented for the other nonphysical attenuation parameters. The advantages of the proposed model for a rigid foundation over a half-plane are shown in comparison to other solutions from the literature, but also to exact analytical result. The importance of well-defined boundary condition is best demonstrated when compared to a model with viscous damper boundary. In fact, even if a perfectly matched layer is a rigorous boundary condition, it performs much better, reducing the computational time in half by using fewer elements. Moreover, an application of the approach is presented where the kinematic effects of vibrating foundation for different ground conditions are evaluated.

1. Introduction

In various fields of engineering, unbounded domains often need to be defined [1]. A typical example can be found in structural dynamics, where the elastodynamic wave equation for an unbounded domain needs to be solved in order to describe the dynamic interaction of a structure and its underlying soil [14]. In particular, the investigation of soil-foundation vibration [25] presents a rather challenging task.

The unbounded domain [24] requires enforcement of a radiation condition in all unbounded direction. Irregularities in the geometry of the domain, or in the material, often require numerical solution by different mathematical formulations, e.g., coupling of the finite element method and the boundary element method [11]. An alternative approach would be to use a discrete element method, i.e., finite element (FE) method in combination with an artificial boundary that absorbs all outgoing waves traveling to infinity [12]. For example the so-called Perfectly Matched Layer (PML) was used as an absorbing layer for linear wave equations. It is reported in literature that it absorbs "almost perfectly, propagating waves of all non-tangential angles-of-incidence and of all non-zero frequencies". The concept was first introduced by Bérenger [5] for electromagnetic waves. Soon after, Chew and Weedon [7,8] showed that Bérenger PML equations arise from a complex valued coordinate stretching in the electromagnetic wave equations. Later, the perfectly matched layer was also formulated for other linear wave equations, namely, the Helmholtz equation [23] and [13], the linearized Euler equations [15] and the wave equation for poroelastic media [26]. Chew and Liu [9] used complex-valued coordinate stretch to obtain equations governing the perfectly matched layer. Basu and Chopra [2] developed displacement based anti-plane, plane-strain and three-dimensional formulations for time-harmonic wave motion as well as for transient wave motion [3]. Moreover, Basu [4] presented a three-dimensional transient finite element implementation of the PML.

This paper focuses on verification and calibration of the time-harmonic elastodynamic FE-PML formulation, which creates an absorbing layer (unphysical domain) around the domain of interest. The absorption property allows an unbounded domain to be modeled by a coupled system of a bounded domain (in the near-field) surrounded by a perfectly matched layer (modeling the far-field). In this study, the PML is combined with a finite element scheme, which is used for simulation of two-dimensional wave propagation. This paper proposes how to improve the PML by

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finding the most efficient attenuation, which controls the performance of the radiation condition.

2. Governing relations

Implementation of the absorbing condition in the elastodynamic equations creates a new set of differential equations that govern the so-called Perfectly Matched Medium (PMM). The process is equivalent to an introduction of a rotated system with stretched coordinates, where the stretching parameter is a complex number. This inhomogeneous medium is later truncated to become a perfectly matched layer. In the present work, a second-order finite element formulation of the perfectly matched layer in frequency domain is perused.

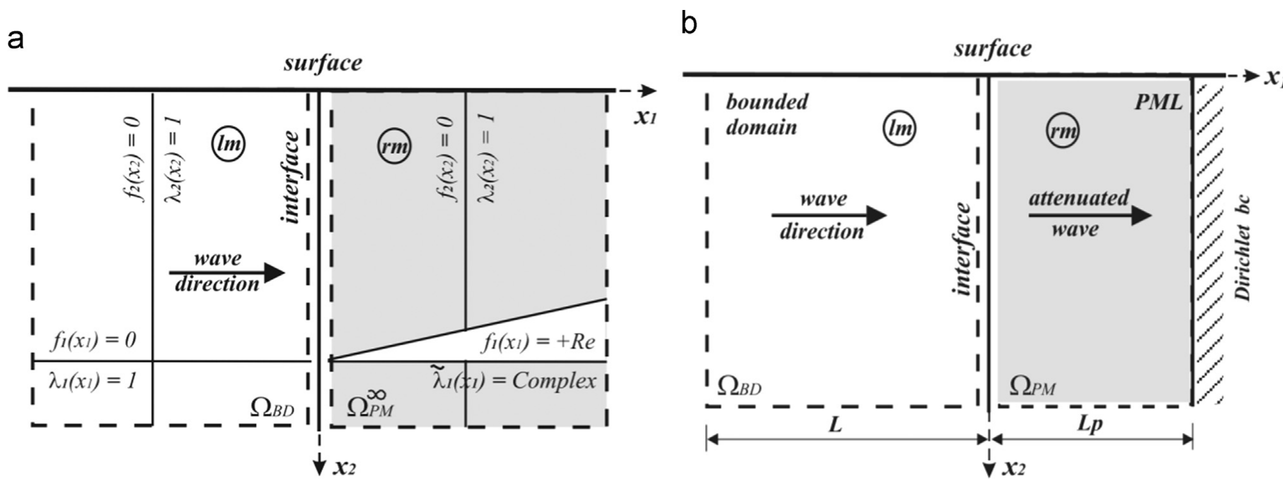

Fig. 1. Definition of dynamic boundary (a) Adjacent PMMs as left (*lm*) and right (*rm*) media (b) PMM truncation with a fixed edge.

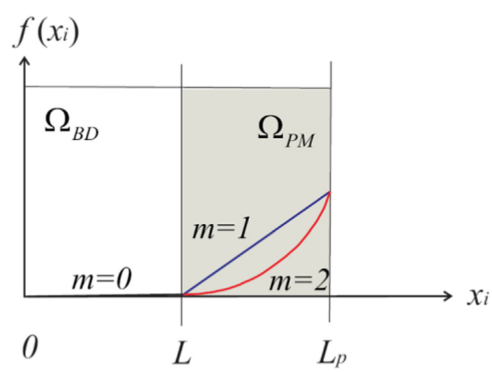

Fig. 2. Attenuation function $f_i(x_i)$.

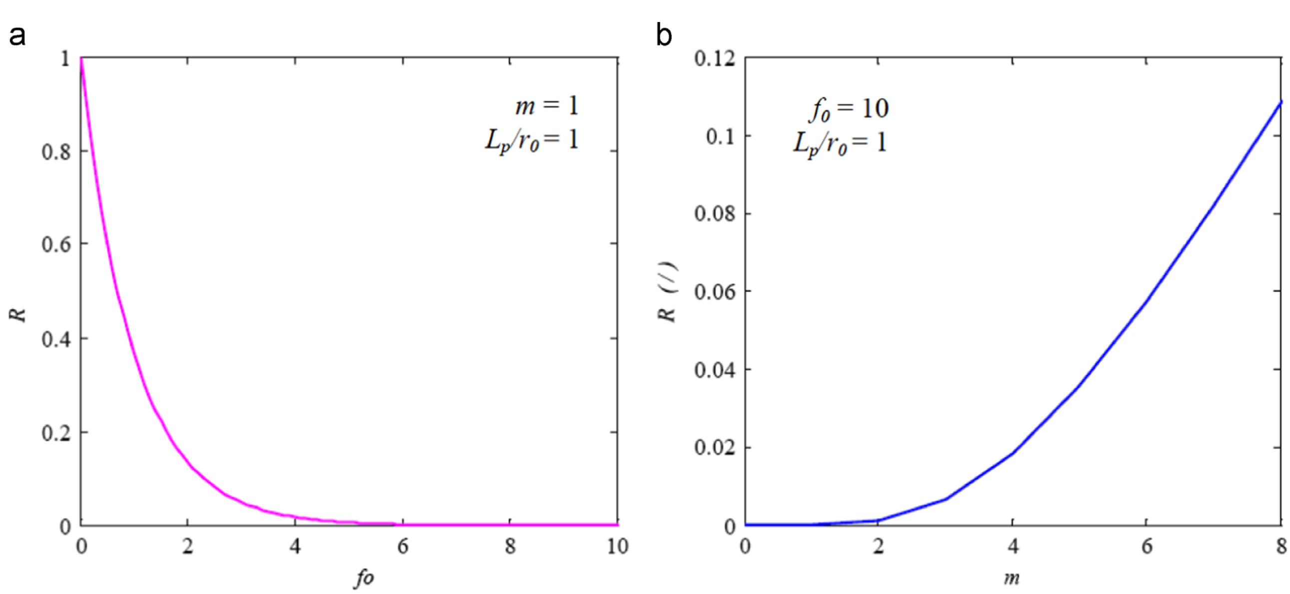

Fig. 3. Reflection coefficient R depending on (a) f_0 and (b) m.