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Analytical solution of the asymmetric transient wave in a transversely isotropic half-space due to both buried and surface impulses

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ABSTRACT

With the aid of a complete set of two scalar potential functions, the problem of transient wave propagation in transversely isotropic half-space, subjected to time dependent tractions applied on a finite patch at an arbitrary depth below the free surface of the half-space is investigated. With the use of the displacement–potential function relationships in a cylindrical coordinate system, the coupled equations of motion are uncoupled; resulting in two separate partial differential equations one of which is second order and the other is fourth order. These two partial differential equations are solved with the aid of both Fourier series expansion and joint Hankel–Laplace integral transforms. The solutions are also investigated in details for tractions varying with time as Heaviside step function, which may be used as a kernel in any integral based method for more complicated elastodynamic initial-boundary value problems. Moreover, some displacement Green's functions are numerically evaluated for a synthetic transversely isotropic material to graphically demonstrate the transient motion of the free surface of the half-space.

1. Introduction

Because of its applications and mathematical challenges, both engineers and mathematicians are interested in the wave propagation in elastic solids especially in a time domain (see for example [1,2]). The study of elastic wave propagation, particularly those with transient nature, has many applications in linear and nonlinear soil–structure-interaction, foundation analysis including piles and underground structures [3], dynamic compaction of soil, dynamic replacement of soil, Earthquake engineering, foundation of theoretical seismology, geophysical related problems and machine foundation design [4–9]. The fundamental solutions for transient elastodynamics of either full-space or half-space may be used for integral base numerical solution of nonlinear soil–structure interaction for more complicated geometry [10,3]. Analytical solutions play an important role in a deep understanding of a scientific phenomenon [11,12], although some simplifications need to be made in the process of deriving them. In particular, analytical solutions can also play a unique role in validating many new numerical methods [13,14]. For these reasons, analytical solutions have been derived in recent years for many scientific problems. In actual engineering problems, where the effects of complex loading situation and complex boundary conditions are indispensable, the numerical methods must be used to solve the problem. One of the powerful numerical methods for solving the linear partial differential equations arise in engineering problems is the boundary element method (BEM), where analytical solution in the domain is, (with the aid of Betti's theorem [4]), obtained after determining the values of the interested fields at the boundary, numerically [10,3]. However, this method needs the determination of the Green's functions for the problem associated with the boundary conditions. Thus in the recent years, a lot of researches have been devoted for determination of Green's functions. Rajapakse and Wang [15], with the use of displacement potential function accompanied with Fourier transform determined the dynamic displacement Green's functions of an orthotropic elastic half-plane subjected to a time-harmonic buried force. Wang and Rajapakse [16] found the internal source Green's function for a transversely isotropic half-space in a time domain in both 2D and 3D cases, where the joint of Laplace–Fourier and Laplace–Hankel integral transforms were used, respectively for 2D and 3D states after using a displacement potential functions for the equations of motion. Wang and Achenbach [17] determined both the 3D and 2D time-domain elastodynamic Green's functions for linearly elastic anisotropic materials with the application of Radon transform. Their fundamental solutions are in the form of a surface integral over the surface of a unit sphere for 3-D cases and

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are over a unit circle path for 2D cases. In addition, their Green's functions are evaluated in the frequency domain readily by a subsequent evaluation of the Fourier transforms of the time-domain solutions. Kausel [18] presented the Green's functions for many different cases such as SH line load, double couples, suddenly line and point loads, etc.

One of the most important contributions in the analytical study of transient wave propagation in elastic isotropic materials is due to Pekeris [19–21]. With the aid of the Laplace and Hankel integral transforms, implementation of Helmholtz decomposition theorem, and the use of Cagniard-De Hoop trick [22,23], Pekeris [21,22] derived an analytical solution for the transient equations of motion in an axisymmetric half-space due to surface and buried impulse loading. In particular, he computed the displacement at the free surface and showed the arrival time of different waves including $P-$, $SV-$ and Rayleigh waves. Chao [24] derived a closed form solution for radial and tangential displacements at the surface of a half-space due to surface horizontal point force varying with time as a Heaviside step function. Jin and Liu [25], with the use of the joint Hankel–Laplace integral transforms accompanied with Cagniard–De Hoop method, have determined the exact analytical solution for the horizontal displacement at the center of a circular surface patch of an elastic isotropic half-space, which is under an impulsive constant distributed loading.

Anisotropy is a common property of engineering materials such as soil (because of sedimentation), rock, reinforced concrete and many man-made materials such as composites and piezo-composites. Thus, the wave propagation in anisotropic materials is recently of major concern. The high performance of anisotropic materials in technological applications is another reason for studying the response of anisotropic material to mechanical force, displacement and other phenomenon. Most innovative materials such as composites, piezo-composites and magnetics are anisotropic, and in applications need to be modeled as either transversely isotropic or orthotropic materials [26,27]. The early work of Stoneley [28] revealed that wave propagation in a transversely isotropic medium gives rise to a phenomenon, which greatly differs from the case where the medium is isotropic. Later, Synge [29], Buchwald [30] and Payton [31] studied the elastodynamic problems pertinent to the transversely isotropic half-space.

The potential method is a powerful tool for solving the coupled both equilibrium equations and equations of motion. Lekhnitskii in 1940 derived a potential function for axisymmetric elastostatic problems of transversely isotropic media [32,33]. Hu [34] and Nowakii [35] studied the general case of elastostatic problem in transversely isotropic media and generalized Lekhnitskii's solution to the asymmetric case, which is now called as Lekhnitskii–Hu–Nowacki solution [36]. Eskandari-Ghadi [33] has introduced a complete solution for the general elastodynamics problems in linear transversely isotropic mono-axial-convex domain in terms of two potential functions, one of which describes *SH*-wave and the other gives both *SV*- and *P*-waves in any plane containing the axis of material symmetry. With the aid of this representation, Eskandari-Ghadi and Sattar [37], investigated the problem of transient wave in an axisymmetric transversely isotropic half-space due to surface loading and their solution included an integral representation with a finite limit.

In the present study, a transversely isotropic half-space is considered as the domain of the problem, and the potential functions introduced by Eskandari-Ghadi [33] is implemented to derive the analytical solution for the displacement Green's function of transversely isotropic half-space under the action of transient tractions applied at an arbitrary depth of the half-space. To do so, with the use of the representations for the displacements, in terms of two scalar potential functions; the elastodynamic governing partial differential equations are uncoupled into a fourth- and a second-order partial differential equations in cylindrical coordinate system and solved by virtue of Fourier series expansion in terms of the angular coordinate and joint Hankel–Laplace integral transforms in term of radial-time variables, along with satisfying both the boundary and regularity conditions.

The Green's functions derived in this paper are applicable as integral kernels in the boundary element method or any other boundary integral formulations to solve more complicated engineering initial-boundary value problems such as either linear or non-linear dynamic analysis of anisotropic soil–structure-interaction as well as earthquake engineering and rock engineering relevant problems. For instance, the topic of the forced vibrations of rigid disc embedded at an arbitrary depth in a semi-infinite transversely isotropic medium, which is a subject of considerable interest in geo-mechanics and civil engineering could be treated with the aid of these Greens' functions [38,39]. Another interesting application of the proposed model may be found in geophysical applications, such as earthquake and volcano source monitoring. Moreover, the Green's functions for the point load excitation may be used in the dislocation formulation of co-seismic deformations arise from the rupture of buried faults, so that they can find some applications in the emerging computational geosciences field [40–43].

2. Statement of the problem

A transversely isotropic half-space in a cylindrical coordinate system is considered as the domain of the problem in such a way that the axis of symmetry of the material to be depth-wise (Fig. 1). The displacement equations of motion in the cylindrical coordinate system for homogenous transversely isotropic solid in the absence of body force may be expressed as [26]:

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+C_{66}\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+C_{44}\frac{\partial^2 u}{\partial z^2}-(C_{11}+C_{66})\frac{1}{r^2}\frac{\partial v}{\partial \theta} +(C_{12}+C_{66})\frac{1}{r}\frac{\partial^2 v}{\partial r\partial \theta}+(C_{13}+C_{44})\frac{\partial^2 w}{\partial r\partial z}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$C_{66}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+C_{11}\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}+C_{44}\frac{\partial^2 v}{\partial z^2}+(C_{13}+C_{44})\frac{\partial^2 w}{\partial \theta\partial z} +(C_{12}+C_{66})\frac{1}{r}\frac{\partial^2 u}{\partial r\partial \theta}+(C_{11}+C_{66})\frac{1}{r^2}\frac{\partial u}{\partial \theta}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{2}$$

$$C_{44}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)+(C_{13}+C_{44})\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}+\frac{1}{r}\frac{\partial^2 v}{\partial \theta\,\partial z}\right) +C_{33}\frac{\partial^2 w}{\partial z^2}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{3}$$

where $\mathbf{u}=(u,v,w)$ is the displacement vector, C_{11}, C_{33}, C_{12}, C_{13}, C_{44} and $C_{66}=(C_{11}-C_{12})/2$ are the elastic constants and ρ is the density of the medium. In view of the positive definiteness of the

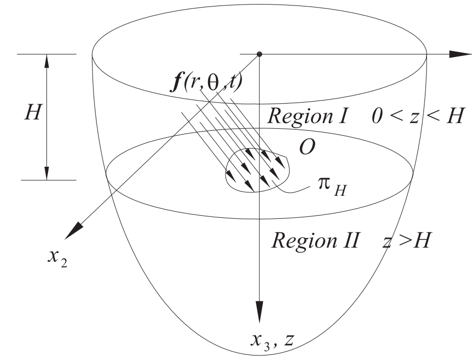

Fig. 1. Transversely isotropic half-space under buried arbitrary time dependent force.