Soil Dynamics and Earthquake Engineering 82 (2016) 142–153

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Seismic pressures on rigid cantilever walls retaining elastic continuously non-homogeneous soil: An exact solution

C. Vrettos [a,*], D.E. Beskos [b,c], T. Triantafyllidis [d]

[a] Division of Soil Mechanics and Foundation Engineering, Technical University of Kaiserslautern, 67663 Kaiserslautern, Germany
[b] Department of Civil Engineering, University of Patras, 26500 Patras, Greece
[c] Office of Theoretical and Applied Mechanics, Academy of Athens, 11527 Athens, Greece
[d] Institute of Soil Mechanics and Rock Mechanics, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany

ARTICLE INFO

Article history:
Received 3 June 2015
Received in revised form
25 November 2015
Accepted 11 December 2015
Available online 5 January 2016

Keywords:
Retaining walls
Rigid cantilever walls
Seismic pressures
Elastic non-homogeneous soil
Analytical–numerical analysis

ABSTRACT

The dynamic response of an elastic continuously nonhomogeneous soil layer over bedrock retained by a pair of rigid cantilever walls to a horizontal seismic motion and the associated seismic pressure acting on these walls are determined analytically–numerically. The soil non-homogeneity is described by a shear modulus increasing nonlinearly with depth. The problem is solved in the frequency domain under conditions of plane strain and its exact solution is obtained analytically. This is accomplished with the aid of Fourier series along the horizontal direction and solution of the resulting system of two ordinary differential equations with variable coefficients by the method of Frobenius in power series. Due to the complexity of the various analytical expressions, the final results are determined numerically. These results include seismic pressures, resultant horizontal forces and bending moments acting on the walls. The solution of the problem involving a single retaining wall can be obtained as a special case by assuming the distance between the two walls to be very large. Results are presented in terms of numerical values and graphs using suitable dimensionless quantities. The effect of soil non-homogeneity on the system response is assessed through comparisons for typical sets of the parameters involved.

1. Introduction

Seismic analysis and design of retaining walls constitute an important area in geotechnical earthquake engineering, which has received considerable attention, especially in recent years. The existing methods for seismically analyzing retaining walls (in almost all cases under plane strain conditions) can be grouped in the following three categories: (a) analytical limit-state analysis methods where the wall can displace and/or rotate sufficiently at its base to induce a limit or failure state in the backfill soil; (b) analytical linear elastic or viscoelastic methods where the wall remains fixed at its base and the backfill soil responds in a linearly elastic or viscoelastic manner; (c) numerical methods of solution, mainly finite element methods (FEM), under the assumption of linear elastic or non-linear elastoplastic soil behavior.

In the first category one can mention the classical Mononobe-Okabe (M-O) method [1,2] and its variants by Seed and Whitman [3], Richards and Elms [4] and Nadim and Whitman [5], which have been adopted by codes of practice [6,7]. Many more methods of this category have been proposed in recent years for improving the original ones.

In the second category of methods one can mention the works of Matsuo and Ohara [8], Wood [9], Scott [10], Arias et al. [11], Veletsos and Younan [12–15], Younan and Veletsos [16], Wu and Finn [17,18], Li [19], Li and Aquilar [20], Jung et al. [21], Papazafeiropoulos and Psarropoulos [22], Kloukinas et al. [23] and Papagiannopoulos et al. [24]. A comparison between the results of the two first categories of methods and those from experiments has been conducted by Giarlelis and Mylonakis [25].

Among the plethora of numerical methods of the third category, mainly FEM, used to analyze the seismic behavior of retaining walls under linear elastic or non-linear elastoplastic soil behavior, one can mention those of Sarfeld et al. [26], Siddharthan and Maragakis [27], Navarro and Samartin [28], Al-Homoud and Whitman [29], Degrande and Aubry [30], Elgamal et al. [31], Wu and Finn [18], Psarropoulos et al. [32], Ostadan [33], Jung et al. [21], Callisto and Soccodato [34], Al Atik and Sitar [35], Evangelista et al. [36], Cakir [37], Argyroudis et al. [38] and Athanasopoulos-Zekkos et al. [39].

Analytical methods of solution, like the ones of the first two categories, are very useful in practice due to their simplicity.

* Corresponding author.
E-mail address: vrettos@rhrk.uni-kl.de (C. Vrettos).

http://dx.doi.org/10.1016/j.soildyn.2015.12.006
0267-7261/

However, they have their advantages and limitations. Even numerical methods of solutions have their limitations, mainly because of using simplified constitutive laws for the soil. Since the present work deals with a solution of the second category, the following discussion is restricted to analytical solutions with emphasis on methods of the second category, i.e. linear elastic or viscoelastic methods of solution.

Elastic methods of solution appeared in the period 1960–1981 [8–11] as an effort of determining the dynamic response of non-yielding soil-wall systems (basement walls or bridge embankments) for which the conditions of limit equilibrium (wall yielding and formation of a soil prism behind it) are not valid, and elastic dynamic pressures are larger than those of the active failure state. During the middle to late 1990's Veletsos and Younan [12–16] studied in depth elastic solutions of the problem of the seismic response of retaining walls, demonstrated their usefulness, and draw the following conclusions: (a) elastic solutions, unlike limit-state solutions, take into account wave propagation in the soil and do not assume a constant acceleration in the backfill; (b) within the range of their applicability, elastic solutions are more rational than limit-state ones, especially with respect to the determination of the base shear force and the point of application of the resultant seismic pressure; (c) for the case of rigid and fixed at their base walls, elastic solutions provide very large wall pressures as compared to those of limit-state ones or tests. However, these solutions are safe upper bound ones and become close to those of tests when the walls are considered as flexible and/or capable of experiencing base rotation [14,16]. Non-homogeneity of the soil [13] and consideration of flexible foundation [19,20] further reduce the elastic dynamic pressures.

In all the aforementioned references concerning the elastic solutions, with the exemption of those in [9,22], the elastodynamic problem consisting of two coupled partial differential equations has been solved with the aid of some reasonable simplifying assumptions, which have resulted in reducing the problem to just one partial differential equation. These assumptions involved mainly either the vertical normal stress or the vertical displacement to be zero everywhere.

Wood [9] and Papazafeiropoulos and Psarropoulos [22] have been able to obtain the exact solution of the problem of the seismic behaviour of a pair of rigid cantilevered walls retaining elastic soil by using the method of modal superposition and Fourier series/solution of ordinary differential equations, respectively. The case of a single wall can be easily obtained by assuming a large distance between the two walls. Availability of exact solutions is invaluable as enabling one to judge the validity of the various simplifying assumptions and determining the degree of approximation. The respective exact solution for water-saturated linear poroelastic soil has been derived very recently by Papagiannopoulos et al. [24]. At this point one should mention the very recent work of Brandenberg et al. [40], who, through a kinematic soil-structure interaction approach, were able to explain both relatively low and relatively high seismic pressures on rigid walls in comparison to Mononobe-Okabe [1,2] predictions for $\lambda/H>10$ and $\lambda/H=4$, respectively, where λ/H is the seismic wavelength to wall height ratio.

In this work, the problem of the seismic behaviour of a pair of rigid cantilever walls retaining elastic soil with a shear modulus nonlinearly varying with depth is solved analytically in an exact manner for the first time in the literature. The problem has been solved analytically in an approximate way by Veletsos and Younan [13] and numerically with the aid of the FEM by Wu and Finn [18] and Psarropoulos et al. [27]. The present solution is obtained by combining the procedures of Papazafeiropoulos and Psarropoulos [22] and Vrettos [41–44]. More specifically, the problem is solved in the frequency domain making use of Fourier series along the horizontal direction [22] and then solving the resulting system of two ordinary differential equations with variable coefficients by the method of Frobenius in power series form [41–44]. Due to the complexity of the analytical expressions, some steps of the analysis are carried out numerically; despite this, the solution can be viewed as exact. The effect of soil non-homogeneity on the response is assessed through comparisons of the results for different parameter sets after introducing suitable dimensionless quantities.

2. Statement of the problem

Consider a pair of rigid cantilever walls retaining a linear elastic soil layer on bedrock with shear modulus G that varies nonlinearly with depth, as shown in Fig. 1. The interface between the bedrock or the walls and the soil is assumed frictionless. The soil-walls system under conditions of plane strain is subjected at its base to a horizontal and space-invariant seismic acceleration $\tilde{a}=\tilde{a}(t)$ which gives rise to in-plane wave motion of the soil medium with displacement components relative to the bedrock base $\tilde{u}(x,z,t)$ and $\tilde{w}(x,z,t)$ along the horizontal (x) and the vertical (z) direction, respectively, as shown in Fig. 1, where t denotes time. The soil domain has a length L and height H, and is characterized by a constant mass density ρ, a Poisson's ratio ν $(0\le\nu<0.5)$ and a shear modulus G varying with depth z according to

$$G(z)=G_0+(G_\infty-G_0)(1-e^{-\alpha z}) \tag{1}$$

where G_0 and G_∞ are the shear moduli at the surface and at infinite depth, respectively, and α is a constant with dimension of inverse length, which is referred to as the non-homogeneity gradient. The above expression was taken from Vrettos [41] who developed it after analysing data obtained from extensive field and laboratory experiments on various granular soils. Note that Eq. (1) is capable of describing both increasing and decreasing soil stiffness with depth. At the level of the bedrock we have $z=H$, and the value of the shear modulus at this position is denoted by G_H. In Eq. (1) G_∞ is a parameter with an asymptotic value, which is never reached in the case of a layer of finite depth.

Assuming a harmonic variation of $\tilde{a}(t)$ with time of the form

$$\tilde{a}=a\exp(i\omega t) \tag{2}$$

where a is the acceleration amplitude, ω is the circular frequency, and i is the imaginary unit, the displacements $\tilde{u}$ and $\tilde{w}$ will be also harmonic of the form

$$\tilde{u}(x,z,t)=u(x,z)\exp(i\omega t) \tag{3}$$

$$\tilde{w}(x,z,t)=w(x,z)\exp(i\omega t) \tag{4}$$

where u and w are their amplitudes.

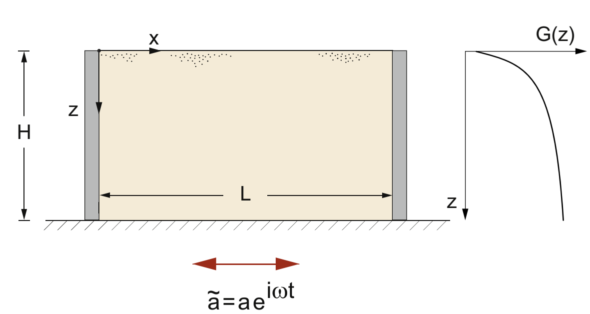

Fig. 1. Geometry of a pair of rigid walls retaining non-homogeneous elastic soil stratum over bedrock under seismic horizontal motion.