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Parametric investigation for rigid circular foundations undergoing vertical and torsional vibrations

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ABSTRACT

This study derives the vertical and torsional responses of a rigid circular foundation resting on unbounded soil. The rigid foundation is subjected to vertical incident *P*-waves, torsional incident waves, vertical harmonic forces, and harmonic torques, respectively. The solutions of the proposed method are found to agree well with existing solutions of previous research. Results of dimensionless parameter analysis show that the vertical response of the rigid surface foundation can be simplified to a function of the Poisson ratio, the dimensionless frequency and mass ratio, and that the torsional response can be further simplified to a function of dimensionless frequency and mass ratio. As the dimensionless mass ratio increases, the vertical and torsional responses increase at low frequencies but decrease at high frequencies, while the fundamental frequency of the foundation-soil system also decreases. As the Poisson ratio increases, the vertical response increases at low frequencies but increases at high frequencies, while the fundamental frequency of the system increases. As the damping ratio of the viscoelastic half-space increases, the vertical and torsional responses rapidly decrease, and the fundamental frequency of the system slightly decreases. The study found significant characteristics of dynamic responses for the foundation undergoing vertical and torsional vibrations, which can provide helpful physical insights for foundation designs.

1. Introduction

The phenomenon of mutual interaction between soil and structures involves two types of mechanisms: inertial interaction and kinematic interaction. The forced vibration analysis for structures only involves inertial interaction. The analysis of seismic wave propagations for structures involves both inertial interaction and kinematic interaction. As a result, seismic wave propagation analysis is more complex than forced vibration analysis.

Modern foundation vibration research originated with the dynamic Boussinesq's problem proposed by Lamb in 1901 [1]. In 1936, Resisner [2] used Lamb's solution to propose a theory regarding a circular foundation resting on a linear elastic half-space subjected to vertical vibration. Resisner in 1937 [3] and Resisner and Sagoci in 1944 [4] solved the response of a rigid circular foundation undergoing torsional vibration. In 1953, Sung [5] and Quinlan [6] continued Resisner's research to analyze the impact of the contacting pressure distribution on the vertical response for circular foundations and rectangular foundations. Lysmer [7] in 1965 and Hsieh [8] in 1962 used a mass–spring–dashpot model, also known as the lumped-parameter models, to analyze the vertical vibration of rigid foundations. In 1971, Luco et al. [9] and Veletsos et al. [10] used the boundary element method to derive the analytical solution for the impedance function of a rigid circular foundation resting on an elastic half-space. In 1972, Novak et al. [11] found an approximate solution for the vertical vibration response of a rigid foundation embedded in an elastic half-space. In 1973 and 1974, Veletsos et al. [12,13] found an approximate solution for the dynamic response of a rigid circular foundation resting on a viscoelastic half-space. In 1986, Veletsos and Tang [14,15] solved analytically the vertical vibration of a rigid circulation foundation resting on a viscoelastic half-space. In 1988 and 1990, Veletsos et al. [16,17] solved the vertical and torsional impedance functions for pile foundations in a viscoelastic half-space. Recently, by using the theoretical impedance functions, Chen and Shi [18–21] have developed a series of lumped-parameter models to simulate the unbounded soil for a foundation-soil system subjected to dynamic forces.

On the other hand, the analysis of seismic wave propagation shall solve the kinematic interaction problems, and therefore the inputs for seismic waves must also consider the scattering problem with different incident wave fields. Once the relationships between various combinations of the dimensionless parameters

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(including mass ratios, excitation frequencies, Poisson ratios, etc.) and the amplification function are solved, then they can be very beneficial for the seismic design of structures. Related works have been presented by Luco et al. [22–26].

Most of existing studies used approximate solutions or numerical solutions to solve impedance functions for rigid massless foundations or to use complicated boundary element methods calculating the dynamic response of foundations. This study directly derives the analytical solutions for the vertical and torsional responses of a rigid circular foundation resting on an elastic or viscoelastic half-space. The rigid foundation is subjected to incident seismic waves (including vertical *P*-waves and torsional waves) and forced vibrations (including vertical harmonic forces and harmonic torques). A parametric study is also conducted to investigate the dynamic characteristics of vibrating responses for the foundation undergoing vertical and torsional motions. The dominated dimensionless parameters for the dynamic foundation response are also clarified and investigated in this study.

2. Development of the equation of motion

This section mainly derives the analytical solutions for the rigid foundation undergoing vertical and torsional vibrations. The vibrating sources come from seismic waves and external forces. The dominated dimensionless parameters for the foundation response are also clearly presented.

2.1. Vertical vibration

As shown in Fig. 1(a), consider that a rigid circular foundation rests on the unbounded soil subjected to a vertical disturbance W_g, also known as an vertical incident *P*-wave. Its equation of motion is:

$$Q_i + Q_s = M\ddot{W}_t + K_z W_0 = 0, \tag{1}$$

where Q_i is the inertia force; Q_s is the reaction force of soil, including spring force and damping force; M is the mass of the foundation; K_z is the vertical impedance function of the foundation; W_t is the total vertical displacement; and W_0 is the relative vertical displacement, also known as the inertial interaction displacement. The total vertical displacement W_t can also be expressed as the following equation:

$$W_t = W_g + W_0, \tag{2}$$

where W_g is the displacement of the foundation base caused by a vertical *P*-wave disturbance, also known as the kinematic interaction displacement. The relationship between W_g and the vertical displacement in a free field, W_f, can theoretically be solved by the scattering problems [22–26]. However, because of the vertical incident *P*-waves, $W_f = W_g$. By substituting Eq. (2) into Eq. (1), Eq. (1) can be rewritten as:

$$M\ddot{W}_0 + K_z W_0 = -M\ddot{W}_g. \tag{3}$$

By using the complex response method, the displacements W_t, W_0, and W_g are assumed to be harmonic vibrations as shown below:

$$W_t = w_t e^{i\omega t}; W_0 = w_0 e^{i\omega t}; W_g = w_g e^{i\omega t}, \tag{4}$$

where ω is the excitation frequency in radian/sec, and w_t, w_0, and w_g are the complex amplitudes of W_t, W_0, and W_g, respectively. By substituting Eq. (4) into Eq. (3), Eq. (3) can be represented as:

$$(K_z - \omega^2 M)w_0 = \omega^2 M w_g. \tag{5}$$

Substituting Eq. (2) into Eq. (5) yields:

$$\frac{w_t}{w_g} = 1 + \frac{\omega^2 M}{K_z - \omega^2 M}. \tag{6}$$

Assuming the soil is an elastic half-space, the vertical impedance function K_z can be expressed as:

$$K_z = kst_z(k_z + i a_0 c_z), \tag{7}$$

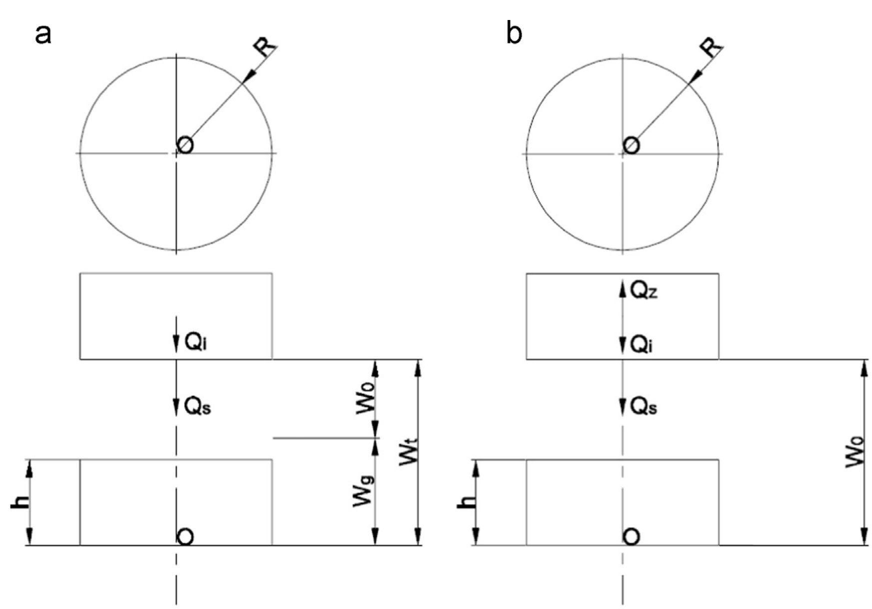
where:

$$kst_z = \frac{4GR}{1-\nu} \tag{8}$$

is the static stiffness:

$$a_0 = \frac{\omega R}{V_s} \tag{9}$$

is the dimensionless frequency; $V_s = \sqrt{G/\rho}$ is the shear wave velocity of the soil; G and ρ are the shear modulus and density of soil; R is the radius of the circular foundation; ν is the Poisson ratio of soil; k_z and c_z are the vertical stiffness coefficient and vertical



Subjected to vertical incident *P*-waves Subjected to harmonic vertical loading

Fig. 1. Vertical vibrations of rigid circular foundations.