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Constitutive modeling of sand: Formulation of a new plasticity approach

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ABSTRACT

A constitutive model for sand is derived based on a new theoretical framework that combines features of perfect elastoplasticity and smooth hysteresis. It resembles a bounding surface model with vanished elastic region, but with considerable modifications in that the plastic modulus is not explicitly defined, and the mapping rule is Bouc–Wen motivated and works equally well in monotonic as in stress-reversal loading. Among the proposed features, are: (a) critical state compatibility not only for monotonic but also for cyclic loading, and (b) novel plastic flow rule accounting for anisotropic distribution of the dilatancy strain ratio, *d*, to the normal plastic strain increments. The capability of the model in capturing complex aspects of sand behavior (e.g. cyclic mobility, static liquefaction, densification) is demonstrated through illustrative paradigms with emphasis on the physical meaning of each key-model parameter and comparisons with experimental data.

1. Introduction

The behavioral diversity of sand for different loadings (drained/undrained, monotonic/cyclic), initial stresses and fabric conditions, renders its modeling a difficult and challenging task. The suitability of the used constitutive model is evaluated by its capability to capture the trends across all these conditions without recalibration of its parameters for each specific case, but also by its simplicity. Too many parameters might increase the versatility of the model at the risk, however, of losing its physical meaning.

In the last three decades, many constitutive models for sand have been proposed, each with varying degree of accuracy and applicability. The most promising ones are plasticity-based that incorporate the effective stress and critical state concepts (e.g.: [5,7,9,14,22,27–31,36]), though recently developed hypoplastic modelshave shown remarkable predictive capability (e.g. [19,23]). In this paper, a constitutive model for sand is presented based on a new plasticity framework that joins together features from perfect elastoplasticity and Bouc–Wen type of hysteresis. The motivation is to develop an alternative plasticity formulation that exhibits critical state compatibility for both monotonic and cyclic loading and uniqueness of its parameters for a given type of sand, irrespective of loading conditions.

The model, designated as Ta-Ger sand model, is based on a reformulation of perfect elastoplasticity by introducing a hardening law inspired from smooth hysteretic modeling. The objective of smooth hysteretic models is to provide a continuous function between an input (displacement, strain etc.) and an output (force, stress etc.) for systems exhibiting hysteresis. In the areas of smart structures and civil engineering, the main class of smooth hysteresis models is represented by the so-called Bouc–Wen model, originally proposed by Bouc [4] and subsequently extended by Wen [41] and used in random vibration analysis of inelastic systems. Since then, modified or extended versions of this model have been extensively applied in modeling structural (e.g. [17,34,39]) and soil behavior (e.g. [15,16,18]). In the present study, the smooth-hysteresis concept is combined with perfect elastoplasticity, resulting in a definite and continuous expression of an elastoplastic matrix, connecting the strain (input) with the stress increment (output). The motivation behind this new plasticity approach, which couples the perfect elastoplasticity with pre-failure smooth hysteresis, is the formulation of an explicit elastoplastic matrix with the following advantages when compared to classical elastoplasticity: (i) the explicit definition of the plastic modulus and the loading index is not necessary, and (ii) no inversion of the strain–stress increment equation is required, since the current plasticity approach is formulated from the start in stress–strain increment terms. Therefore, stress point algorithms

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and iterative procedures associated with the prediction of the loading index and the stress increment, are avoided, rendering the numerical implementation a simpler and more efficient task.

The framework of the new plasticity approach is then used for developing a constitutive model for sand. The developed constitutive formulation can be regarded as a single-surface model with vanished elastic region and the distinguished characteristic of an explicitly defined plastic matrix instead of a plastic modulus. Salient features of the proposed plasticity approach are: (i) a new plastic flow rule which is based on a revision of Rowe's dilatancy theory (1962) to account for anisotropic distribution of the dilatancy to the normal plastic strain increments as well as densification due to cyclic loading, (ii) a mapping rule incorporated in the elastoplastic matrix and a load reversal criterion based on the sign of the first order work, and (iii) a new formulation for the critical state concept that introduces two "state" parameters. The first being the cumulative incremental deviatoric strain, controlling the transition from the initial to the critical state strength ratio and the second one being the relative dilatancy index, I_R, as originally proposed by Bolton [3], associated with the critical state line in $D_r - p$ space. The main advantage against the traditional formulation involving only the state parameter Ψ [1,27], is critical state consistency for cyclic loading and avoidance of early shear locking in cyclic undrained response (as observed in [9]).

In the following sections, the reader is initially introduced to the proposed plasticity approach which is a general platform that can be used for a macroscopic constitutive description of a wide range of hysteretic materials when combined with an appropriate failure surface and plastic flow rule. The development of the new plasticity framework is presented step by step resulting in the formulation of an elastoplastic matrix that involves two key-variables, ζ and n. Without loss of generality, the role of each key-variable in the current plasticity framework is exemplary demonstrated by using the Drucker–Prager failure surface. The presented plasticity platform, serves as the basis for developing an integrated constitutive model for sand. The formulation of the constitutive model is described in detail, emphasizing the effect of the key-variables and state parameters on the behavior of sand. Finally, the model is shown to be capable of reproducing complicated experimental behavior with satisfactory engineering accuracy, however, its complete calibration is beyond the scope of this paper.

2. Plasticity concept: combining perfect plasticity with Bouc–Wen type hysteresis

The governing equations of a typical elastoplastic formulation with no hardening (elastic/perfectly-plastic behavior), in generalized stress space, are revisited. For reasons of simplicity and convenience, the equations are given in the form of matrices instead of tensors in this section. The incremental total strain, $\{d\epsilon\}$, is decomposed into its elastic and plastic counterparts $\{d\epsilon^e\}$ and $\{d\epsilon^p\}$:

$$\{d\epsilon\} = \{d\epsilon^e\} + \{d\epsilon^p\} \tag{1}$$

The plastic strain increment is obtained from the flow rule:

$$\{d\epsilon^p\} = \langle l \rangle \frac{\partial g(\{\sigma\})}{\partial \{\sigma\}} \tag{2}$$

Eq. (2) applies normality of the plastic strain increment to a plastic potential function g. l is the scalar-valued stress-dependent multiplier, designated as the loading index. Substituting Eq. (2) into Eq. (1) and applying the theory of elasticity, $\{d\sigma\} = \{E^e\}\{d\epsilon^e\}$, $\{E^e\}$ being the linear elastic matrix, the following stress–strain relationship is

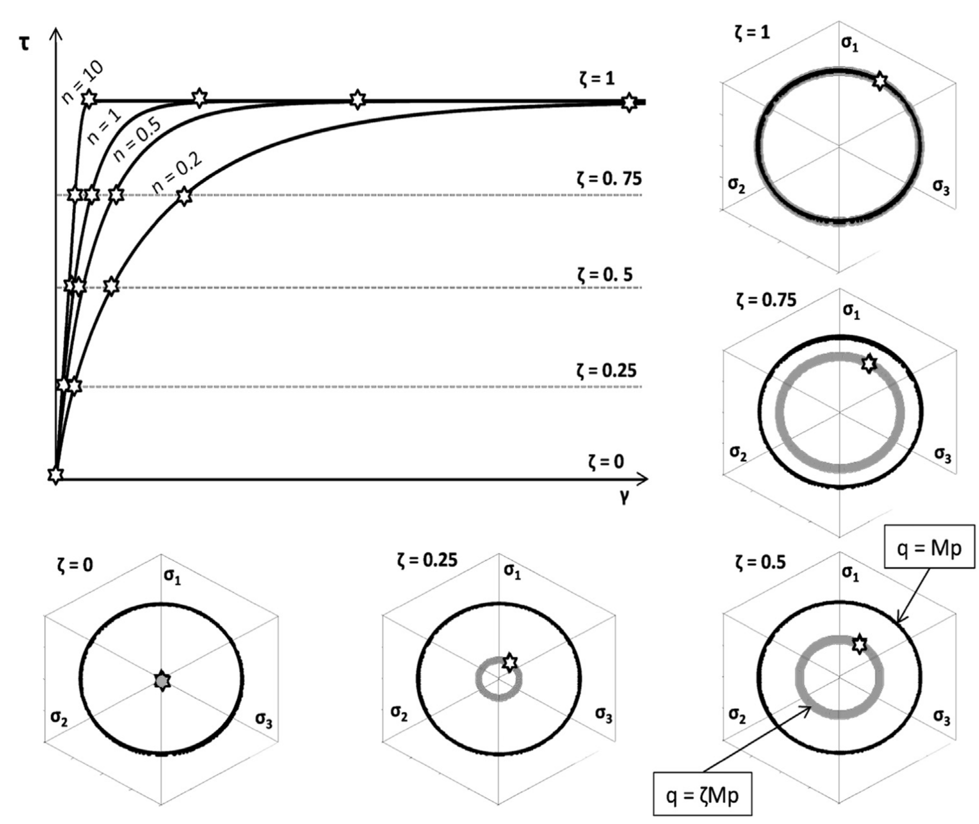

Fig. 1. Evolution of parameter ζ during loading and influence of hardening exponent n on the predicted response, in case of a Drucker–Prager failure criterion. The star symbols characterize the current stress states. The π-plane plots correspond to $n = 1$.