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Technical Note

An analytical model to predict the natural frequency of offshore wind turbines on three-spring flexible foundations using two different beam models

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ABSTRACT

In this study an analytical model of offshore wind turbines (OWTs) supported on flexible foundation is presented to provide a fast and reasonably accurate natural frequency estimation suitable for preliminary design or verification of Finite Element calculations. Previous research modelled the problem using Euler–Bernoulli beam model where the foundation is represented by two springs (lateral and rotational). In contrast, this study improves on previous efforts by incorporating a cross-coupling stiffness thereby modelling the foundation using three springs. Furthermore, this study also derives the natural frequency using Timoshenko beam model by including rotary inertia and shear deformation. The results of the proposed model are also compared with measured values of the natural frequency of four OWTs obtained from the literature. The results show that the Timoshenko beam model does not improve the results significantly and the slender beam assumption may be sufficient. The cross-coupling spring term has a significant effect on the natural frequency therefore needs to be included in the analysis. The model predicts the natural frequency of existing turbines with reasonable accuracy.

1. Introduction

In order to ensure optimum performance throughout its design life, predicting the long term behaviour of offshore wind turbines (OWTs) is essential. However, data are scarce on the long term performance of these complex mechanical systems. The loading of OWTs is complex due to a combination of static, cyclic and dynamic loads [1]. However OWTs must be designed to avoid forcing frequencies due to wind turbulence, waves and also the rotational frequency (1 P) and the blade passing frequencies. The importance of dynamics for offshore wind turbines is well established in the literature ([1–5]) and it is well known from the literature that repeated cyclic or dynamic loads ([12–14]) on a soil can cause a change in its properties leading to alteration of the stiffness of the foundation ([2,6,7]). A wind turbine structure derives its stiffness from the supporting foundation and any change in its stiffness may shift the natural frequency closer to the forcing frequencies. This issue is particularly problematic to the soft-stiff structure (natural frequency between 1 P and 3 P frequency) as any increase or decrease in the natural frequency will impinge on the forcing frequencies and may lead to unplanned resonance and increased fatigue damage. This may lead to loss of years of service, which is to be avoided.

Difference between design and measured natural frequency is reported in the literature. Two examples are considered here: (a) *Walney 1 Wind farm*: the actual natural frequency was 6–7% higher than the estimated for a Siemens SWT-3.6-107 turbine at the Walney 1 site, see [9]; (b) *Twisted jacket at Hornsea Site*: difference between the design and measured frequency was observed in the case of the Hornsea Met Mast supported on a 'Twisted Jacket' foundation [8]. In this demonstration project it was found that the foundation was stiffer than expected and the initial measured frequency was 1.28–1.32 Hz as opposed to the design frequency of 1 Hz. Furthermore, after three months, the natural frequency shifted to 1.13–1.15 Hz, likely due to softening of the soil. These cases clearly highlight the importance of prediction of the natural frequency.

The aim of this work is to provide an analytical estimation for the natural frequency of monopile supported offshore wind turbines where the foundation is modelled using three springs: (a) Lateral spring (K_L); (b) Rotational spring (K_R); (c) A cross coupling spring (K_{LR}) which is in contrast to the uncoupled springs model ([1,3,9,10]). Furthermore, present study also extends the analysis by incorporating the Timoshenko beam model ([11,12]) which also accounts for rotary inertia and shear deformation.

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2. Structural model of the offshore wind turbine

The structural model used in this paper is shown in Fig. 1. The foundation is represented by three springs: lateral K_L, rotational K_R and cross K_{LR} stiffness. The tower is idealised by equivalent bending stiffness and mass per length following [9,13] and is modelled using two beam theories: Euler–Bernoulli and Timoshenko. The later accounts for shear deformation and the effect of rotational inertia. The nacelle and rotor assembly is modelled as a top head mass with mass moment of inertia.

2.1. Foundation model

In Fig. 1 the foundation is represented by four springs, a lateral K_L, a rotational K_R, a cross coupling K_{LR} and also a vertical spring (K_V), which is neglected because the structure is very stiff vertically. The method of Gazetas [14] can be used for the estimation of the spring stiffness of slender piles (also recommended in Eurocode 8, Part 5 [15]), however, this is not validated for very large diameter piles. In the absence of directly measured values of stiffness, the Finite Element (FE) approach may produce more reliable results (see e.g. Lesny et al. [16]). The three spring model can be written with a stiffness matrix as the following:

$$\begin{bmatrix} F_x \\ M_y \end{bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{LR} & K_R \end{bmatrix} \begin{bmatrix} w \\ w' \end{bmatrix} \tag{1}$$

where F_x is the lateral force, M_y is the fore-aft moment, w is the displacement and $w' = \partial w / \partial z$ is the slope.

2.2. Model of the rotor-nacelle assembly

The rotor-nacelle assembly is modelled as a top head mass M_2 with mass moment of inertia J, as shown in Fig. 1. These parameters are used in formulating the end boundary conditions of the PDEs of the motion of the tower in Section 2.3. In addition, the mass M_2 exerts a downwards pointing force P due to gravity, and the self-weight of the structure also acts on the sections below. The total vertical force is

$$P = -M_2 g - mg(L - z) \tag{2}$$

where m is the average mass per length of the tower, L is the height of the tower. An approximate expression for a constant

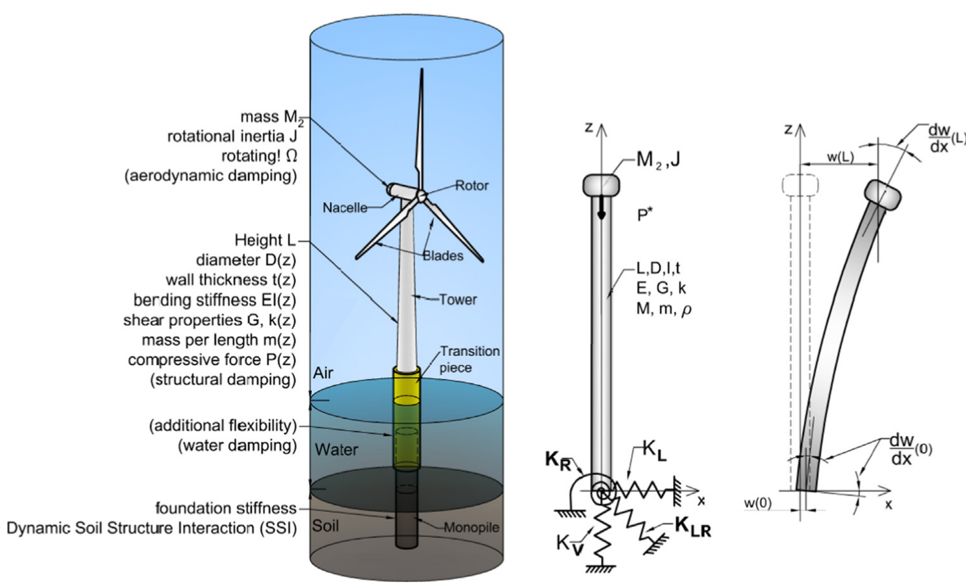

Fig. 1. Mechanical model of a wind turbine.

Table 1
Non-dimensional groups: definitions and practical range.

Dimensionless group	Formula	Typical values
Non-dimensional lateral stiffness	$\eta_L = K_L L^3/EI$	2500–12000
Non-dimensional rotational stiffness	$\eta_R = K_R L/EI$	25–80
Non-dimensional cross stiffness	$\eta_{LR} = K_{LR} L^2/EI$	(−515) to (−60)
Non-dimensional axial force	$\nu = P^* L^2/EI$	0.005–0.1
Mass ratio	$\alpha = M_2/M_3$	0.75–1.2
Non-dimensional rotary inertia	$\beta = J/mL^2$	[a]
Non-dimensional shear parameter	$\gamma = E/Gk$	~4.5 (for steel tubular towers)
Non-dimensional radius of gyration	$\mu = r/L$	
Frequency scaling parameter	$c_0 = \sqrt{EI/M_3 L^3}$	~1–5
Non-dimensional rotational frequency	$\Omega_k = \omega_k / c_0 = \omega_k \sqrt{EI/M_3 L^3}$	–

K_L, K_R, K_{LR} are the lateral, rotational and cross stiffness of the foundation, respectively; EI is the equivalent bending; L is the height of the tower; P^* is the modified axial force (see Eq. 3), M_2 is the top head mass; M_3 is the mass of the tower; J is the rotary inertia of the top mass; m is the equivalent mass per unit length of the tower; $r = \sqrt{I/A}$ is the radius of gyration of the tower, ω_k is the kth natural frequency.

[a] The rotary inertia is taken to be zero for all wind turbines considered as information is not available in the referenced literature.