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Plane strain dynamic response of a transversely isotropic multilayered half-plane

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ABSTRACT

A semi-analytical method is developed to analyze the plane strain dynamic response of a transversely isotropic multilayered half-plane subjected to a time-harmonic surface or buried load. On the basis of the governing equations of motion in Cartesian coordinates, the analytical layer-elements of a single layer with a finite thickness and a half-plane are obtained through the Fourier transform and the corresponding algebraic operations. The analytical layer-element solution for the multilayered half-plane in the transformed domain can be derived in combination with the continuity conditions between two adjacent layers. After the boundary conditions are introduced, the corresponding solution in the frequency domain is recovered by the inverse Fourier transform. The comparison with an existing solution for an isotropic half-plane confirms the accuracy of the proposed method. Several examples are given to portray the influence of material anisotropy, the depth of external load, material stratification and the frequency of excitation on the vertical displacement and vertical normal stress.

1. Introduction

The research of dynamic response problems is of great importance to the studies related to elastic wave propagation in soils caused by external loadings such as transportation, machine and pile driving. As the pioneer, Lamb [1] first studied the response of an isotropic elastic half-space subjected to a time-harmonic surface load. The follow-up studies for dynamic response problems dealing with an isotropic half-space or full-space can be seen in references (Achenbach [2], Miklowitz [3], Pak [4], Pak and Ji [5], Guzina and Pak [6], etc.). Scholars (Aspel and Luco [7], Pak and Guzina [8], Xu et al. [9], etc.) got analytical solutions for dynamic response of an isotropic multilayered half-space.

However, soils in geotechnical engineering are generally transversely isotropic due to long-term sedimentation processes. Many researches, as in Pan and Chou [10,11], Yue et al. [12], Liao and Wang [13], and Wang and Liao [14] provided fundamental solutions for a transversely isotropic half-space subjected to different kinds of static loadings. Apart from transversely isotropy, soils also take on the phenomenon of layering. Solutions for a transversely isotropic multilayered medium under static loads can be found in references (Small and Booker [15,16], Singh [17], Pan [18,19], Ai et al. [20], etc.). Therefore, it is more realistic to regard soils as a transversely isotropic multilayered medium and to study theirs dynamic response problems.

Regarding the dynamic response of a transversely isotropic medium, Stoneley [21] was the earliest researcher focusing on wave propagation in a transversely isotropic medium. Synge [22] and Buchwald [23] studied the propagation of Rayleigh waves in a transversely isotropic medium. Payton [24] presented a time domain solution for displacements and stresses in a transversely isotropic full-space loaded by an instantaneously applied point force. Later, Payton [25] summarized the dynamic problems of a transversely isotropic elastic half-space under surface loads in his book published in 1983. Rajapakse and Wang [26,27] gave out the Green's functions for a 2-D transversely isotropic half-plane and a non-axisymmetrical transversely isotropic half-space subjected to an interior time-harmonic load. The 3-D time harmonic Green's function for a transversely isotropic medium was given by Zhu [28] and Yang et al. [29], respectively. Eskandri-Ghadi [30] introduced two potential functions as a general solution for a transversely isotropic medium. With the aid of the potential functions presented by Eskandri-Ghadi [30], Rahimian et al. [31] and Khojasteh et al. [32,33] achieved more subsequent studies in dynamic response problems of a transversely isotropic medium.

As for a multilayered system, Khojasteh et al. [34] used the method of displacement potentials to obtain 3-D dynamic Green's

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functions in a two-layered transversely isotropic half-space. Later, they [35] extended the 3-D dynamic Green's functions to a transversely isotropic multilayered medium. Ai et al. [36], Ai and Li [37] utilized the analytical layer-element method to derive the solutions for a transversely isotropic multilayered half-space under vertical and horizontal time-harmonic loads, respectively. From above, it can be found that most of the researches concern about a transversely isotropic half-space, while the researches of a multilayered system, especially the ones in Cartesian coordinates, are quite limited. In practical engineering, strip foundations, embankments and dams are generally considered as the plane strain problems, which is more suitable for Cartesian coordinates rather than cylindrical coordinates, therefore the dynamic response of a multilayered transversely isotropic medium in Cartesian coordinates should receive more attention.

The purpose of this paper is to extend the analytical layer-element method to solve the plane strain dynamic response of a transversely isotropic multilayered half-plane subjected to a time-harmonic surface or buried load. Compared with the work of Refs. [34,35], the presented method only involves negative exponentials functions in the stiffness matrices of soils, which not only simplifies the calculation processes but also improves the numerical efficiency and stability. With the application of the Fourier transform and the corresponding algebraic operations, the analytical layer-elements which describe the relationship between stresses and displacements of a single layer with a finite thickness and a half-plane are obtained. According to the continuity conditions between adjacent layers, the global stiffness matrix equation is further achieved. After the boundary conditions are introduced, the solution in the frequency domain is achieved by taking the inversion of the Fourier transform. Selected numerical results are performed to demonstrate the accuracy of present method, and to discuss the influence of material anisotropy, material stratification, the depth of load and the frequency of excitation.

2. The analytical layer-elements for a single layer and a half-plane

In a Cartesian coordinate system, defined that the z-axis is normal to the plane of isotropy, the governing equations of motion in the absence of body forces for an elastic body can be expressed as follows:

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=\rho\frac{\partial^2 u_x}{\partial t^2} \tag{1a}$$

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \sigma_z}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2} \tag{1b}$$

where σ_x and σ_z represent the normal stress components in the x and z directions, respectively; τ_{xz} stands for the shear stress component in the planes xz; u_x and u_z are the displacement components in the x and z directions, respectively; ρ denotes the density of the material; t is the time variable.

The constitutive equations of a transversely isotropic body, which have five independent elastic parameters, can be written in terms of displacements as follows:

$$\sigma_x=c_{11}\frac{\partial u_x}{\partial x}+c_{13}\frac{\partial u_z}{\partial z} \tag{2a}$$

$$\sigma_y=c_{12}\frac{\partial u_x}{\partial x}+c_{13}\frac{\partial u_z}{\partial z} \tag{2b}$$

$$\sigma_z=c_{13}\frac{\partial u_x}{\partial x}+c_{33}\frac{\partial u_z}{\partial z} \tag{2c}$$

$$\tau_{xz}=c_{44}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right) \tag{2d}$$

where $c_{11}=\lambda n(1-n\mu_{vh}^2)$, $c_{12}=\lambda n(\mu_h+n\mu_{vh}^2)$, $c_{13}=\lambda n\mu_{vh}(1+\mu_h)$, $c_{33}=\lambda(1-\mu_h^2)$ and $c_{44}=G_v$ are the five independent elastic parameters, in which $n=E_h/E_v$, $\lambda=E_v/[(1+\mu_h)(1-\mu_h-2n\mu_{vh}^2)]$. Here, E_v, E_h and G_v are the vertical Young's modulus, horizontal Young's modulus and shear modulus, respectively. In addition, μ_{vh} and μ_h are Poisson's ratios characterizing horizontal strain due to parallel and normal stresses acting on the plane, respectively.

We assume the load is time-harmonic with the circular frequency ω, so the displacement components may express in the form of $u_x(x,z,t)=u_x(x,z)e^{i\omega t}$ and $u_z(x,z,t)=u_z(x,z)e^{i\omega t}$, and the harmonic time factor $e^{i\omega t}$ is suppressed.

Substitution of Eqs. (2) into Eqs. (1) leads to the following equations:

$$c_{11}\frac{\partial^2 u_x}{\partial x^2}+c_{44}\frac{\partial^2 u_x}{\partial z^2}+(c_{13}+c_{44})\frac{\partial^2 u_z}{\partial x\partial z}=\rho\frac{\partial^2 u_x}{\partial t^2} \tag{3a}$$

$$(c_{13}+c_{44})\frac{\partial^2 u_x}{\partial x\partial z}+c_{33}\frac{\partial^2 u_z}{\partial z^2}+c_{44}\frac{\partial^2 u_z}{\partial x^2}=\rho\frac{\partial^2 u_z}{\partial t^2} \tag{3b}$$

The integral transformation approaches are employed to reduce the partial differential equations mentioned above into ordinary differential equations. According to Sneddon [38], a Fourier integral transform is taken. The Fourier transform with respect to the variable x and its inversion are defined as

$$(\overline{u}_x,\overline{u}_z,\overline{\sigma}_z,\overline{\tau}_{xz})=\frac{1}{2\pi}\int_{-\infty}^{+\infty}(iu_x,u_z,\sigma_z,i\tau_{xz})e^{-i\xi x}dx \tag{4a}$$

$$(u_x,u_z,\sigma_z,\tau_{xz})=\int_{-\infty}^{+\infty}(-i\overline{u}_x,\overline{u}_z,\overline{\sigma}_z,-i\overline{\tau}_{xz})e^{i\xi x}d\xi \tag{4b}$$

where ξ is the Fourier transform parameter with respect to the variable x, and $i=\sqrt{-1}$.

Eqs. (3) are treated by the Fourier transform Eq. (4a), then we have:

$$\left(\rho\omega^2-\xi^2c_{11}+c_{44}\frac{d^2}{dz^2}\right)\overline{u}_x-(c_{13}+c_{44})\xi\frac{d\overline{u}_z}{dz}=0 \tag{5a}$$

$$(c_{44}+c_{13})\xi\frac{d\overline{u}_x}{dz}+\left(\rho\omega^2-\xi^2c_{44}+c_{33}\frac{d^2}{dz^2}\right)\overline{u}_z=0 \tag{5b}$$

Eqs. (5) may be recast into:

$$\frac{d^2\overline{u}_x}{dz^2}=\frac{(c_{13}+c_{44})}{c_{44}}\xi\frac{d\overline{u}_z}{dz}-\frac{\left(\rho\omega^2-\xi^2c_{11}\right)}{c_{44}}\overline{u}_x \tag{6a}$$

$$\frac{d^2\overline{u}_z}{dz^2}=-\frac{(c_{44}+c_{13})}{c_{33}}\xi\frac{d\overline{u}_x}{dz}-\frac{\left(\rho\omega^2-\xi^2c_{44}\right)}{c_{33}}\overline{u}_z \tag{6b}$$

In order to simplify the analysis, several variables are defined as follows:

$$\overline{\boldsymbol{W}}(\xi,z)=\left[\overline{\mathbf{U}}(\xi,z),\overline{\boldsymbol{U}}'(\xi,z)\right]^{T} \tag{7a}$$

$$\overline{\boldsymbol{U}}(\xi,z)=[\overline{u}_x(\xi,z),\overline{u}_z(\xi,z)]^{T} \tag{7b}$$

$$\overline{\boldsymbol{U}}'(\xi,z)=\left[\frac{d\overline{u}_x(\xi,z)}{dz},\frac{d\overline{u}_z(\xi,z)}{dz}\right]^{T} \tag{7c}$$

With the aid of Eqs. (7), Eqs. (6) take the following form:

$$\frac{d\overline{\boldsymbol{W}}(\xi,z)}{dz}=\mathbf{A}(\xi)\overline{\boldsymbol{W}}(\xi,z) \tag{8}$$