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Wave propagation in soils problems using the Generalized Finite Difference Method

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ABSTRACT

The consideration of highly irregular topography or heterogeneities and singularities in the domain are necessary for solving a wide variety of seismic problems, e. g. earthquake site studies. Therefore, the use of a meshless method (MM) with the possibility of employing an irregular grid point distribution can be of interest for modeling this kind of problems. The tradition of using Finite Different Methods for modeling seismic wave propagation problems have allowed researches to solve it and, in fact, a numerical model using the geotechnical calculation software FLAC has been used here to validate the results. It is clear the improvement of using meshless method techniques to incorporate irregularities in a natural way.

1. Introduction

During recent years, meshless methods have emerged as a class of effective numerical methods which are capable of avoiding the difficulties encountered in conventional computational mesh-based methods. An important path in the evolution of meshless methods has been the development of the Generalized Finite Difference Method (GFDM), also called meshless finite difference method [1]. The bases of the GFDM were published in the early seventies. Jensen [2] was the first author to introduce a fully arbitrary mesh. He considered Taylor's series expansions interpolated on six-node stars in order to derive the Finite Difference (FD) formulae, approximating derivatives of up to the second order. Perrone and Kao [3] suggested that additional nodes in the six-point scheme should be considered and an averaging out process for the generalization of finite difference with coefficients applied. The idea of using an eight node star and weighting functions, to obtain finite difference formulae for irregular meshes, was first put forward by Liszka and Orkisz [4] using moving least squares (MLS) interpolation, and an advanced version of the GFDM was given [1]. Benito et al. [5] studied the dependency of solution of the Generalized Finite Difference Method on the number of nodes in the cloud, the relative coordinates of the nodes with respect to the star node and on the weight function employed. An h-adaptive method in GFDM is described in [6–8].

The application of GFDM to the solution of the problem of seismic wave propagation using PML absorbing boundary was introduced by Ureña et al. [9]. These authors have also applied this meshless method to the solution of dynamic problems of beams and plates [10], to solve the advection–diffusion equation [11] and to solve parabolic and hyperbolic equations [12].

Absorbing boundary condition must be used in numerical wave problems to truncate unbounded media without the reflection due to numerical boundaries, and Perfectly Matched Layer (PML) is one of the most effective among them. This method has been widely used for Finite Differences Methods (FDM).

Berenger [13] first created a PML for electromagnetic problems in a Finite Element (FE) numerical model. Their equations are based on a field splitting which results in mathematical expressions that can not be easily manipulated. Chew and Weedou (1994), following Berenger's work, introduced the concept of using complex coordinates in the formulation. Sacks et al. (1995) developed an anisotropic PML also valid to the Finite Element Method (FEM).

Chew and Liu [14] first proposed the PML for elastic waves in solids and proved that reflections are null in a regular elastic medium. PML has become very successful in many fields, and in the context of wave propagation: the PML has been applied to acoustic by Qi and Geers (1998), Hagstrom (1999), Liu and Tao (1997) and Kormann et al. (2008) to underwater acoustic propagation models; it has been applied to wave propagation in poroelastic media by Zeng and Huang [15]; to

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elastic problems by Colino and Monk (1998), Colino and Tsogka [16], Basu and Chopra [17]; FE applied to classical soil-structure interaction problems in the frequency domain, Cohen and Fauqueux (2003) and with a FDM scheme Moczo et al. [18], Kirsch [19], Skelton et al. [20], Johnson [21], and for wave equation written as a second-order system in displacements by Komatitsch and Tremp [22] and Benito et al. [23].

This paper shows the application of GFDM to solve these kinds of problems. The scheme used and the analyses of stability and dispersion have been clearly referenced. The use of PML absorbing boundary conditions in the model is also explained and the influence of the loss profile is shown. Finally, the efficiency of the method in solving a variation of Lamb's problem on a domain with a simple topographical feature is illustrated as well as another one including a hole.

2. Explicit generalized differences schemes for the seismic wave propagation problem: second order formulation

The equations of motion for a perfectly elastic, homogeneous, isotropic medium in the domain $\Omega \subset R^2$ are

$$\rho U_{i,tt} = (\lambda+\mu)U_{j,ji} + \mu U_{i,kk} \tag{1}$$

where U_i are the components of the displacement, ρ is the density, λ and μ are the Lamé elastic coefficients.

In this paper three types of boundary conditions are considered: homogeneous, Dirichlet boundary conditions, free surface and, in some cases, a symmetry.

On the free surface the following conditions are imposed

$$\sigma_{ij}n_j = (\lambda\ \varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij})n_j = g_i(t) \Leftrightarrow \quad (\lambda\ U_{k,k}\delta_{ij} + \mu(U_{i,j} + U_{j,i}))n_j = g_i(t) \tag{2}$$

where $g_i(t)$ is equal to zero when there are no forces on the surface.

The aim is to obtain explicit linear expressions for the approximation of partial derivatives at the points of the domain. First of all, an irregular grid or cloud of points is generated in the domain $\Omega \cup \Gamma$, where Γ is the boundary of the domain. On defining the central node with a set of nodes surrounding that node, the star then refers to a group of established nodes in relation to a central node. Every node in the domain has an associated star assigned to it. This scheme uses the central-difference form for the time derivative.

Following [1,5,6,8,10], the explicit finite difference formulae for the second spatial derivatives with second and fourth order approximation (p=2, 4) for the spatial derivatives are obtained

$$[U^0_{i,jk}]_{t=n\Delta t} = -m^{0,p}_{jk}\ {}^n u^0_i + \sum_{l=1}^{N} m^{l,p}_{jk}\ {}^n u^l_i + \Theta[(h_i)^p] \tag{3}$$

where capital letters are used for exact values and small letters are used for approximated values. The superscript n denotes the time step, the superscripts 0 and l refer to the central node and the rest of star's nodes respectively, N is the number of nodes in the star (in this work, for the second order approximation N=8 and the star nodes are selected by using the distance criteria [3], and the fourth order approximation N=30 and the star nodes are selected by using the distance criteria) and $h^l_i = x^l_i - x^0_i$.

$m^{0,p}_{jk}$ are the coefficients that multiply the approximate values of the functions U_i at the central node (${}^n u^0_i$) for the time $n\Delta t$ in the generalized finite difference explicit expressions for the space derivatives.

$m^{l,p}_{jk}$ are the coefficients that multiply the approximate values of the functions U_i at the other nodes of the star (${}^n u^l_i$) for the time $n\Delta t$ in the generalized finite difference explicit expressions for the space derivatives. In all these expression the cross-terms are equal.

The replacement in Eq. (1) of the explicit expressions obtained for the spatial derivatives and the central-difference formula for the time derivatives, leads to the explicit difference scheme

$$ {}^{n+1}u^0_i = 2\ {}^n u^0_i - {}^{n-1}u^0_i + \frac{(\Delta t)^2}{\rho}\left[(\lambda+\mu)\left(-m^{0,p}_{ji}\ {}^n u^0_j + \sum_{l=1}^{N} m^{l,p}_{ji}\ {}^n u^l_j\right) + \mu\left(-m^{0,p}_{kk}\ {}^n u^0_i + \sum_{l=1}^{N} m^{l,p}_{kk}\ {}^n u^l_i\right)\right] + \Theta\left[(\Delta t)^2, (h_i)^p\right] \tag{4}$$

Imposing boundary conditions is not difficult except for the case of free surface. The values of the function in the nodes of the free surface are unknown but free surface conditions Eq.(2) are known. If U^{se}_i are the displacements of the n_{se} added nodes (Newmann nodes) that belong to a star being U^s_i the displacement in its central node and U^{si}_i the displacements of the remaining n_{si} nodes of the star. Substituting the first order derivatives that appear in the free surface condition Eq. (2) by the explicit expressions Eq. (3) the system of 2n equations is obtained

$$\left[\lambda\left(-m^s_k\ {}^n u^s_k + \sum_{l=1}^{N} m^l_k\ {}^n u^l_k\right) + \mu\left(\left(-m^s_j\ {}^n u^s_i + \sum_{l=1}^{N} m^l_j\ {}^n u^l_i\right) + \left(-m^s_i\ {}^n u^s_j + \sum_{l=1}^{N} m^l_i\ {}^n u^l_j\right)\right)\right] n_j = g_i(t) \tag{5}$$

where $N = n_{se} + n_{si}$ and the 2n unknowns are the displacements in the n added nodes. These unknowns appear in the summation. By solving this system, the function values (${}^n u^{se}_i$) are obtained on the n added Neumann nodes at the time $t = n\Delta t$.

As an explicit method is being used, the stability has been studied and the star stability condition has been obtained in [9]. In this paper, the condition for stability of the star has been established as:

$$\Delta t < \sqrt{\frac{4}{(\alpha^2+\beta^2)[(|m^{0,p}_{11}| + |m^{0,p}_{22}|) + \sqrt{(m^0_{11} + m^{0,p}_{22})^2 + (m^{0,p}_{12})^2}]}} \tag{6}$$

where

$$\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}; \quad \beta = \sqrt{\frac{\mu}{\rho}}$$

are the velocities of P and S waves respectively.

The dispersion for the phase and group velocities has also been studied in depth in [9].

3. Benchmark test

To analyse the obtained approximation, the method has been applied to the solution of Eq. (1) in a square of unitary length. We have used models with 441 and 1681 nodes, Young's modulus λ=0.5, a shear modulus μ=0.25, a density ρ=1 and Dirichlet boundary conditions at every side. The exact solution is

$$\begin{cases} U_x(x,y,t) = \cos\left(\sqrt{\frac{2\mu}{\rho}}\ t\right)\sin x\ \ \sin y \\ U_y(x,y,t) = \cos\left(\sqrt{\frac{2\mu}{\rho}}\ t\right)\cos x\ \ \cos y \end{cases} \tag{7}$$

Fig. 1 shows the exact solution and the values obtained using the GFD scheme at the node with coordinates (0.5, 0.5) for 500 time steps that are equivalent to 0.5 s. The solution has been obtained with the schemes with second and fourth order approximation for the space second order derivatives.

To validate the results obtained by the application of the GFDM a numerical model using the geotechnical calculation software FLAC [24] is done. An elastic medium of horizontal dimension, in