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A brief theory and computing of seismic ground rotations for structural analyses

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ABSTRACT

The seismic ground rotations are important with respect to spatial structural models, which are sensitive to the wave propagation. The rotational ground motion can lead to significant increasing of structural response, instability and unusual damages of buildings. Currently, the seismic analyses often take into account the rocking and torsion motions separately using artificial accelerograms. We present an exact analytical method, proposed by Nazarov [15] for computing of three rotational accelerograms simultaneously from given translational records. The method is based on spectral representation in the form of Fourier amplitude spectra of seismic waves, corresponding to the given three-component translational accelerogram. The composition, directions and properties of seismic waves are previously determined in the form of a generalized wave model of ground motion. It is supposed that seismic ground motion can be composed by superposition of P, SV, SH- and surface waves. As an example, the dynamic response analysis of 25-story building is presented. Here recorded (low-frequency) and artificial (high-frequency) accelerograms were used; each of them includes three translational and three rotational components. In this structural analysis, we have clarified primarily conditions under which rotational ground motion should be taken into account. Next, we have calculated three rotational components of seismic ground motion. Then they were taken as additional seismic loads components for further seismic analysis of the building. Note, soil–structure interaction (SSI) is not considered in this study. For computing, we use the special software for structural analyses and accelerogram processing (FEA Software STARK ES and Odyssey software, Eurosoft Co., Russia). It was developed and is used in engineering practice in the Central Research Institute of Building Constructions (TsNIISK, Moscow, Russia).

1. Introduction

It is very important that experimental experiences in the field of rotational seismology tend to rise [1]. We hope that in the future, reasonable recorded six-component data will help to progress in applied structural analyses. Meanwhile, engineers mainly use available theoretical methods based on the pioneering work of Newmark [2]. Newmark proposed first relationships between torsional and translational components of ground motion. His approach was applied and advanced by many researchers [2–12]: Luco [3], Shibata et al. [4,5], Lee and Trifunac [6–11], Todorovska et al. [12] and many others; in USSR and Russia by Rasskazovsky [13], Hachiyan [14], Nikolaenko and Nazarov [15–17]. Today, the method of Lee and Trifunac [6,7] to generate the artificial torsional and rocking accelerograms is widely used in structural engineering practice (for an example, see [18]). In Russia, other approach is applied. It is based on the exact relationships between spectra of translational and rotational components and takes into account available recorded data. Before describing this approach, consider the types of seismic ground motions, for which the account of rotations is really necessary.

A contribution of seismic rotations to total ground motion can be evaluated using the value of dominant wavelengths λ. We can introduce approximate differentiation of ground motion types based on the computing experiences. Denoted by B the minimal horizontal dimension of the building foundation. So, if $\lambda > 10B$, corresponding wave field under the building foundation is dilatational (Fig. 1); all shear strains are small. In this case, rotational components are not considered.

If $kB < \lambda < 10B$, where k varies from 2 to 4, this wave field is dilatational–rotational (Fig. 1). Dilatational–rotational seismic ground motion is characterized by both translational and rotational ground

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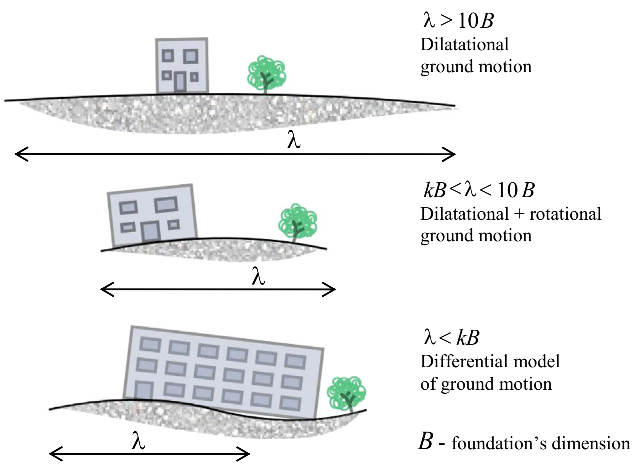

Fig. 1. Types of seismic ground motions.

movements. Consequently, the ground motion is described by a six-component vector, including three translational and three rotational components.

These two models of seismic ground motion (dilatational and dilatational–rotational) are called integrated, because seismic motions are averaged over ground volume and applied at one point as the time-depended vector of seismic accelerations. Note that the averaging is correct only for rigid foundations. Dimensions of the ground volume V must be comparable to dominant wavelengths λ. M.D. Trifunac has introduced a similar definition in [9] about "average rotations (rotation of a line connecting two moving points and separated by a distance that can be comparable to and longer than the representative wavelengths)".

In all other cases, it is impossible to apply the integrated model of the seismic ground motion (Fig. 1). These wave fields depend significantly on space coordinates. In this case, the differential model is used. Seismic ground motions are the vector field defined at each point of space.

To determine dominant wavelengths of seismic waves, we use diagrams of normalized intensities. Consider Fourier spectra of translational ground acceleration $\ddot{X}(t)$. If the function $\ddot{X}$ has period $2T$, its finite Fourier decomposition includes N terms and can be written as

$$\ddot{X}(t) = \sum_{k=0}^{N} A_k \cos\left(k\frac{\pi t}{T} + \varphi_k\right),$$

where A_k are spectral amplitudes, φ_k are phase angles. k-th term corresponds to a simple wave with angular frequency $\omega_k = k\pi/T$ and wavelength $\lambda_k = 2\pi c/\omega_k$, where c is velocity of seismic wave propagation. We can filter out simple waves with wavelengths not exceeding certain value $\bar{\lambda}$. In other words, $\bar{\lambda}$ is the shortest wavelength of filtered process $\ddot{X}$. Denoted by $\sigma_{\ddot{X}}$ the intensity of random process $\ddot{X}$ ($\sigma_{\ddot{X}}$ is equal to standard of $\ddot{X}$), $\sigma_{\ddot{X}}(\bar{\lambda})$ is the intensity of filtered process $\ddot{X}$. The normalized intensity $\chi_1(\bar{\lambda})$ of the translational movement is defined as

$$\chi_1(\bar{\lambda}) = \frac{\sigma_{\ddot{X}}(\bar{\lambda})}{\sigma_{\ddot{X}}}.$$

The $\chi_1(\bar{\lambda})$-diagram shows, what part of total intensity of translational movement relates to wavelengths, which are higher $\bar{\lambda}$.

For an example, consider $\chi_1(\bar{\lambda})$-diagram in Fig. 2 with following input data: recorded accelerogram of Spitak earthquake (1988), direction Z, phase velocity is equal to 100 m/s. Here $\chi_1(250) = 0.15$,

Fig. 2. The dimensionless $\chi_1(\bar{\lambda})$-diagram (computed by Odyssey), $\chi_1(250) = 0.15$.

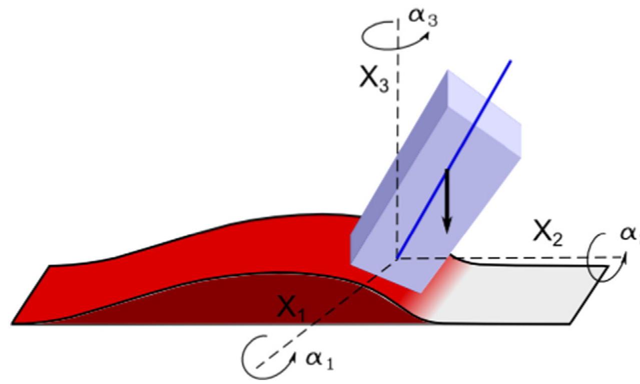

Fig. 3. Seismic rotational angles.

this means that wavelengths, which are higher than 250 m, take only 15% of the total intensity. Therefore, wavelengths up to about $\lambda = 250$ m are dominant.

2. Theory

2.1. Definition of seismic rotations

The wave passage effect can produce both translational and rotational motions about each of the three global axes. Seismic rotational components are angles α_1, α_2, α_3, angular velocities $\dot{\alpha}_1$, $\dot{\alpha}_2$, $\dot{\alpha}_3$ and angular accelerations $\ddot{\alpha}_1$, $\ddot{\alpha}_2$, $\ddot{\alpha}_3$ about global axes (Fig. 3), $\dot{\alpha}_i = d\alpha_i/dt$, $\ddot{\alpha}_i = d^2\alpha_i/dt^2$.

The translational movements $X_i = X_i(x_1, x_2, x_3, t)$ describe a three-dimensional vector field $\mathbf{X}$. A curl of the field $\mathbf{X}$ at the point (x_1, x_2, x_3) is defined by a vector

$$\operatorname{curl}\ \mathbf{X} = \begin{vmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \mathbf{j}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ X_1 & X_2 & X_3 \end{vmatrix},$$

where $\mathbf{j}_1$, $\mathbf{j}_2$, $\mathbf{j}_3$ are unit vectors of corresponding axes. The curl $\mathbf{X}$ coordinates yield the required rotational angles $\boldsymbol{\alpha} = (\alpha_1,\ \alpha_2,\ \alpha_3)^T$:

$$\boldsymbol{\alpha} = \tfrac{1}{2}\operatorname{rot}\mathbf{U},$$

$$\alpha_1 = \frac{1}{2}\left(\frac{\partial X_3}{\partial x_2} - \frac{\partial X_2}{\partial x_3}\right),\quad \alpha_2 = \frac{1}{2}\left(\frac{\partial X_1}{\partial x_3} - \frac{\partial X_3}{\partial x_1}\right),\quad \alpha_3 = \frac{1}{2}\left(\frac{\partial X_2}{\partial x_1} - \frac{\partial X_1}{\partial x_2}\right). \qquad (1)$$

The time derivatives (1) yield angular velocities $\dot{\alpha}_1$, $\dot{\alpha}_2$, $\dot{\alpha}_3$ and angular accelerations $\ddot{\alpha}_1$, $\ddot{\alpha}_2$, $\ddot{\alpha}_3$ at the point (x_1, x_2, x_3):

$$\dot{\alpha}_1 = \frac{1}{2}\left(\frac{\partial \dot{X}_3}{\partial x_2} - \frac{\partial \dot{X}_2}{\partial x_3}\right),\quad \dot{\alpha}_2 = \frac{1}{2}\left(\frac{\partial \dot{X}_1}{\partial x_3} - \frac{\partial \dot{X}_3}{\partial x_1}\right),\quad \dot{\alpha}_3 = \frac{1}{2}\left(\frac{\partial \dot{X}_2}{\partial x_1} - \frac{\partial \dot{X}_1}{\partial x_2}\right). \qquad (2)$$