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Technical Note

The influence of the degree of saturation on dynamic response of a cylindrical lined cavity in a nearly saturated medium

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ABSTRACT

In most previous studies on the dynamic response of a long cylindrical cavity subjected to internal transient dynamic loads, the porous medium was usually assumed to be completely saturated by ground water. In practice, however, the full saturation condition does not always exist. In this paper the surrounding soil and the lining of the cavity are respectively treated as a nearly saturated porous medium and an elastic material, and the governing equations for the dynamic problem are derived. A set of exact solutions are obtained in the Laplace transform domain for three types of transient loads, i.e. suddenly applied constant load, gradually applied step load and triangular pulse load. By utilizing a reliable numerical method of inverse Laplace transforms, the time-domain solutions are then presented. The influence of the degree of saturation of the surrounding soil on the dynamic response of the lined cavity is examined for numerical examples.

1. Introduction

Evaluation of the response of a cylindrical cavity subjected to internal transient loads is a classical dynamic problem in geotechnical engineering. A large number of studies have dealt with the analytical solution on the dynamic problem. These previous studies can be classified into two types. One is to treat the soil surrounding the cavity as an elastic solid so as to simplify the problem (e.g. Refs. [1,2]). The other type focuses on dynamic response of saturated soil surrounding a cavity under internal transient loading. In these studies soil was assumed to be a poroelastic material composed of a solid matrix with fluid and Biot's theory was used (e.g. Refs. [3–6]). In reality soil is a three-phased material, comprising solid particles, water and air. Even for soil under water table the condition of full saturation may not exist (Ref. [7]).

In this paper, therefore, the effect of the degree of saturation on the response of a lined cylindrical cavity under various types of transient loading is investigated on the basis of Biot's theory and the assumption of homogenous pore fluid. The lining is treated as an elastic medium and the one-dimensional wave equation is established for the lining. A set of exact solutions on the response of the lining and the surrounding poroelastic soil is derived in the Laplace transform domain. The method of Durbin [8] is then used to transform the Laplace space solutions into the time-domain solutions. Numerical examples are given to illustrate the influence of the degree of saturation.

2. Theoretical solutions

A cylindrical lined cavity subjected to internal transient loads is presented herein, as shown in Fig. 1. Based on the second Newton's Law, the equilibrium equation for the surrounding soil can be written as

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=-\rho\frac{\partial^2 u_r}{\partial t^2}+\rho_f S_r\frac{\partial^2 w_r}{\partial t^2} \tag{1}$$

Where σ_{rr} and $\sigma_{\theta\theta}$ are the radial and hoop total stresses, respectively; u_r and w_r are respectively the radial displacements of solid skeleton and pore fluid with respect to solid skeleton. ρ_f and ρ are respectively the mass density of pore fluid and soil; S_r is the degree of saturation.

For the pore fluid, the equilibrium equation can be written as

$$\frac{\partial\sigma_f}{\partial r}=\frac{-\rho_f}{n}\frac{\partial^2 w_r}{\partial t^2}-\rho_f\frac{\partial^2 u_r}{\partial t}-\frac{\eta}{k_s}\frac{\partial w_r}{\partial t} \tag{2}$$

where σ_f is the absolute pore fluid pressure at the interface between the lining and the soil; η is the coefficient of viscosity of the fluid; n and k_s are the permeability and porosity of soil, respectively.

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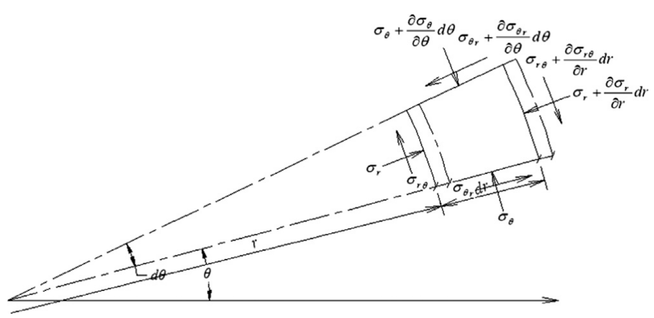

Fig. 1. Diagram of forces acting on an element in soil around a cylindrical lined cavity.

On the basis of the constitutive equations and the strain-displacement relations, the governing equations for the soil and fluid can be written as

$$G_s\left(\nabla^2-\frac{1}{r}\right)u_r+(\lambda_s+\alpha^2 M+G_s)\frac{\partial e}{\partial r}-\alpha M\frac{\partial \xi}{\partial r}=\rho\frac{\partial^2 u_r}{\partial t^2}+\rho_f S_r\frac{\partial^2 w_r}{\partial t^2} \tag{3}$$

$$\alpha M\frac{\partial e}{\partial r}-M\frac{\partial \xi}{\partial r}=\frac{\rho_f}{n}\frac{\partial^2 w_r}{\partial t^2}+\rho_f\frac{\partial^2 u_r}{\partial t^2}+\frac{\eta}{k_s}\frac{\partial w_r}{\partial t} \tag{4}$$

where λ_s and G_s are Lamé constants; α and M are Biot's coefficients; and the variables e and ξ are defined as follows $e=(\partial u_r/\partial r)+u_r/r$, $\xi=-(\partial w_r/\partial r+w_r/r)$.

The governing equations of lining are shown in Ref. [4]. The solutions in Laplace transform domain can then be obtained based on procedures from Gao et al. [4]. Note that if introducing a S_r=100% into the solutions for radial displacement, hoop stress and pore pressure, the proposed solutions exactly degenerate to the ones given by Ref. [3].

3. Results and discussion

The effect of degree of saturation (S_r) of the surrounding soil on dynamic response of the lined cavity is now investigated based on the solutions derived. The parameters used in the calculations are the same as Ref. [3].

Fig. 2 depicts the time history of radial displacement at the cavity surface with respect to the degree of saturation for the three types of transient loads, i.e. suddenly applied constant load, gradually applied step load and triangular pulse load. In Fig. 2 $u(a)$ is the radial displacement at cavity surface; a is the radius of the cavity; f_0 is the maximum dynamic loading amplitude at the inner surface of the lined cavity; t^* is the normalized time [4]. It is seen that there is much less impact on the radial displacement when S_r is less than 99%, but when S_r is larger than 99%, it has a significant effect on the peak value of the radial displacement. The solutions for the case of a suddenly applied constant load are found to be very close to those for the case of a gradually applied step load. The peak value of the radial displacement goes up with an increase of the degree of saturation, while the radial displacement at higher degree of saturation needs more time to reach the peak value, meaning that the degree of saturation significantly affects the velocity of propagation of the dilatational waves. This result is consistent with that in Ref. [9].

Fig. 3 shows the time history of hoop stress at the cavity surface at various degrees of saturation under the three types of transient loads, in which $\sigma_{\theta\theta}(a)$ is the hoop stress at cavity surface. It is noted that the peak values of the hoop stress corresponding to the three types of loads increases with the degree of saturation. However, the effect of saturation is more significant in the latter two load cases. It is also found that the impact of degree of saturation on the hoop stress is less for S_r varying from 95% to 99% in comparison with S_r varying from 99% to 100%.

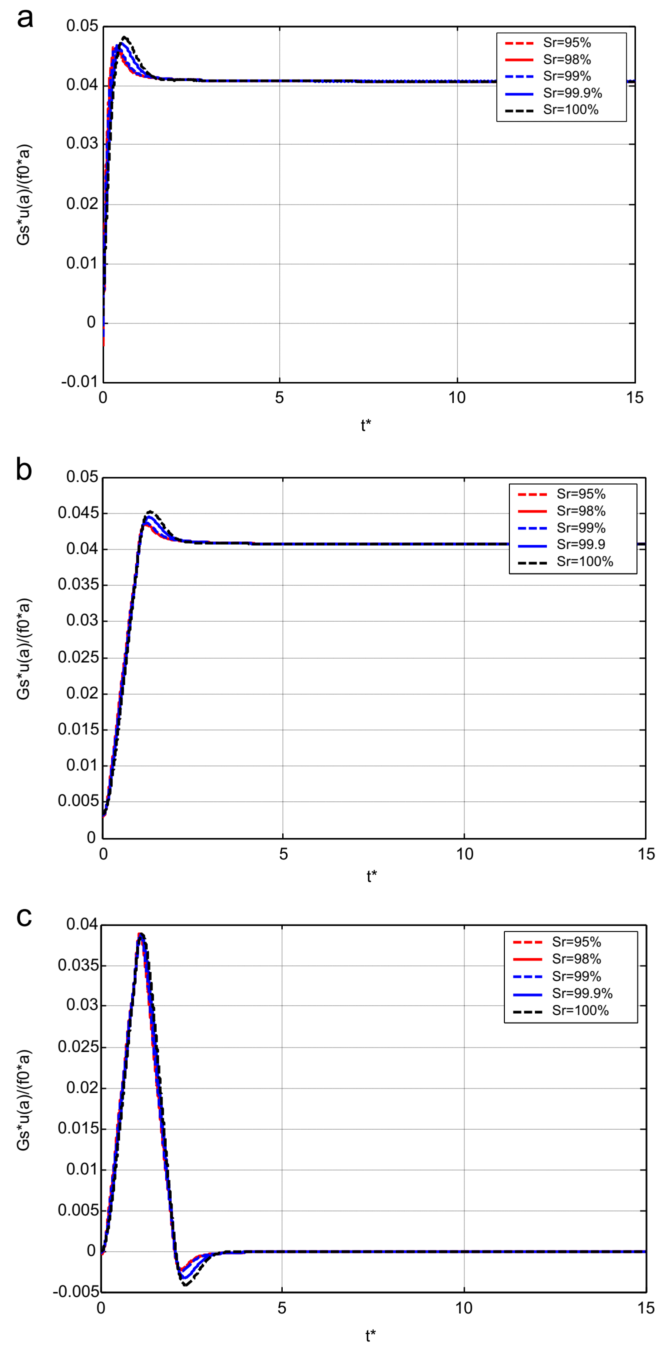

Fig. 2. The time history of radial displacement at the cavity surface for various degrees of saturation for three types of transient loads. (a) Sudden constant load (b) Gradually applied step load and (c) Triangular pulse load.

Fig. 4 shows the time history of pore pressure at the interface between the soil and the lining at various degrees of saturation, in which $\sigma_f(a+h)$ is the absolute pore fluid pressure at the interface between the lining and the soil; h is the thickness of the lining. It can be seen from Fig. 4(a) that the pore pressure drops sharply with time. Fig. 4(b) shows that two peak values exist in a very short time for the case of gradually applied step load. In Fig. 4(c) three peak values are observed for the case of triangular pulse load. It is noted that an increase of degree of saturation leads to a higher peak value and to a delayed arrival of the peak value.