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# Elastic waves in continuous and discontinuous geological media by boundary integral equation methods: A review

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ABSTRACT

In this review paper, we concentrate on the use of boundary integral equation (BIE) based methods for the numerical modeling of elastic wave motion in naturally occurring media. The main reason for using BIE is the presence of the free surface of the earth, whereby large categories of problems involve continua with a small surface to volume ratio. Given that under most circumstances, BIE require surface discretization only, substantial savings can be realized in terms of the size of the mesh resulting from the discretization procedure as compared to domain-type numerical methods. We note that this is not necessarily the case with man-made materials that have finite boundaries. Thus, although the emphasis here is on wave motion in geological media, this review is potentially of interest to researchers working in other scientific fields such as material science. Most of the material referenced in this reviews drawn from research work conducted in the last fifteen years, i.e., since the year 2000, but for reasons of completeness reference is made to seminal papers and books dating since the early 1970s. Furthermore, we include here methods other than the BIE-based ones, in order to better explain all the constituent parts of hybrid methods. These have become quite popular in recent years because they seem to combine the best features of surface-only discretization techniques with those of domain type approaches such as finite elements and finite differences. The result is a more rounded approach to the subject of elastic wave motion, which is the underlying foundation of all problems that have to do with time-dependent phenomena in solids.



## 1. Introduction

Recent developments in the numerical modeling of elastic wave phenomena in naturally occurring geomaterials (i.e., the general inhomogeneous and anisotropic soil medium with layered structure and containing fissures plus various types of discontinuities such as solid inclusions, cavities and tunnels) can be found in numerous scientific journals on computational mechanics, fracture mechanics, geophysics and geomechanics. We note that from a mathematical viewpoint, this numerical modeling effort is applicable to many categories of man-made materials, which implies that further information can be gleaned from material science journals as well. In essence, elastic wave motion in complex media is nowadays modeled using a variety of semi-analytical, numerical and hybrid methods. In reference to the first category, commonly used methods are wave function expansion techniques (Pao and Mow [255]), ray theory and its modifications (Babich [26]; Pao and Gajewski [254]), various matrix equation methods, reflectivity methods, wave number integration methods (Luco and Apsel [198]; Apsel and Luco [21]; Wuttke [366]) and finally mode matching techniques (Fah [107]; Panza et al. [253]). In general, all these approaches are restricted to special types of inhomogeneous media with simple geometries plus a heterogeneity length scale that is considerably larger than the predominant wavelengths.

Generally speaking, numerical techniques are now thought of as being the preferred way forstudying elastic wave motion in inhomogeneous media. Although they are suitable for analyzing media with complex structure, they require much computational effort in terms of both run-times and memory space. Among currently used numerical approaches, BIE methods (BIEM) have become quite popular over the last decade because of their efficient handling of infinite and semi-infinite domains connected with the modeling of the earth (Beskos [39,40]; Manolis and Beskos [207]; Manolis and Davies [209]; Dominguez [93]; Beskos [41]; Zhang et al. [379]; Bonnet [47]; Aliabadi [11]). Furthermore, the BIEM has also been used for modeling buried structures and structural components such as foundations, tunnels, trenches, cavities, etc. (Beskos [42]). This is so because the BIEM allows for the following: (a) Reduction in the problem dimensionality because only surfaces need to be represented, in contrast to domain-methods; (b) direct modeling of lateral inhomogeneity and anisotropy

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through the use of appropriate fundamental solutions or Green's functions; (c) solution at selected internal points that is expressed in terms of boundary-only information and without recourse todomain discretization; (d) good quality numerical results since quadrature techniquesare directly applied to the boundary integral equations, which in turn are an exact mathematical statements for the problem under consideration. This last advantage extends to the modeling of infinite or semi-infinite domains because Sommerfeld's radiation condition is automatically satisfied, without the need to construct special viscous boundaries. All this gives the BIEM a marked advantage over domain methods such as the finite difference method (FDM) and the finite element method (FEM). In addition, the BIEM is well suited for problems where stress gradients are present as for instance in fracture mechanics. Finally, dynamic analysis using the BIEM in both time and frequency domains is possible, provided the proper fundamental solutions (or Green's functions) are available.

The aim of this review article is to present the last BIEM developments to a wide audience ranging from geophysicists to material scientists. The review is organized as follows: (a) Section 2 considers the BIEM for elastic wave propagation in continuous media. We focus here on numerical formulations for elastodynamics, conditional on the availability of suitable fundamental solutions or Green's functions. Next, we present elastic wave scattering problems in homogeneous, continuously inhomogeneous and poroelastic media. (b) Section 3 deals with elastic wave propagation in discontinuous media, i.e., media that contain cracks (or fissures). As before, we first consider BIEM formulations and hybrid numerical techniques for cracks. This is followed with an extensive list of results drawn from the literature for problems of elastic wave propagation in fractured homogeneous isotropic media, homogeneous anisotropic media, layered media and continuously inhomogeneous media. Finally, a set of conclusions is given in Section 4.

## 2. Elastic waves in continuous media

### 2.1. Boundary integral equation formulations for elastodynamics

There are two basic avenues for extending BIE formulations to the various cases that arise when time-dependent behavior of either natural media or manufactured materials is sought: (a) use of specialized fundamental solutions or Green's functions (Melnikov [229]) so that the chief advantage of the BEM, namely surface discretization only, is preserved and (b) use of volume discretization by converting the difference between the actual mechanical state and an ideal state that corresponds to linear elastic, homogeneous and isotropic conditions, into a body force. This is known as the dual-reciprocity boundary element method (DR-BEM) (Cheng et al. [62], Partridge et al. [257]). The resulting volume integral is subsequently computed with the help of specialized interpolation (or basis) functions so that relatively few interior nodes are required for maintaining good numerical accuracy.

BIE-based formulations using elastodynamic fundamental solutions are based on either the direct time-domain approach or the transformed-domain approach. In the latter case, time dependence is removed by taking a Fourier, Laplace or any other transform with respect to the time variable. Thus, the original hyperbolic partial differential equations of motion are reduced to elliptic partial differential equations. The main disadvantage of the latter approach is that non-linear behavior cannot be modeled. In general, the time domain boundary element method (TD-BEM) is a purely space-time formulation since both kinds of integrations, i.e., spatial and temporal, are present. Two basic strategies regarding discretization have been used to date. The most popular one is to define spatial interpolation functions independently from the temporal interpolation functions. The second approach is to define shape that handle spatial and temporal variables in a coupled fashion, see Frangi [113]. Note that we refer here to TD-BEM formulations that utilize fundamental solutions corresponding to Dirac's delta function as input, in lieu of fundamental solutions defined for a step impulse (the Heaviside function), see Karabalis and Beskos [160], Manolis et al. [205] and Rizos and Karabalis [279] among others.

The importance of the early time-domain boundary element methodologies (TD-BEM) for numerically studying wave propagation and other transient problems in mechanics is well recognized (Manolis [202]; Manolis and Ahmad [204]; Mansur and Delima-Silva [223]). The TD-BEM is characterized by high accuracy, as compared to commonly used numerical methods such as the FEM, and is also well suited for problems involving unbounded domains where radiation conditions are correctly satisfied. A drawback of the TD-BEM is that unstable behavior may occur under certain conditions and possibly in an unpredictable way. This type of behavior manifested is sometimes called 'intermittent instability' after Pierce and Siebrits [262]. In the last decade or so, intense research effort has been oriented towards elimination, or at least reduction, of the unstable behavior of TD-BEM formulations. More specifically, the conventional displacement based TD-BEM suffers loss of accuracy for high Poisson's ratio values. Furthermore, when tracking the evolution of 'error' over long time intervals, suddenly instability appears, see Schanz et al. [299]. As one way to remedy this, Schanz [293–295] proposed a TD-BEM that uses the quadrature formula of Lubich [197] for approximating the Riemannconvolution integrals. This approach utilizes Laplace transform domain fundamental solutions instead of the time-domain ones, and requires the continuous switching from one domain to the other in the time stepping numerical scheme.

Other methods that have been introduced can be classified as follows: (a) the theta-method (Yu et al. [373]; Araujo et al. [22]), which adopts the main idea behind the well-known Wilson $\theta$-method used in the FEM, whereby the dependent field variable is predicted at a time interval greater than the time step and then interpolated backwards; (b) the epsilon-method, which may be interpreted as a perturbation of the trapezoidal rule in order to provide improved stability characteristics (Pierce and Siebrits [263]); and (c) the so-called 'half-step' scheme, again proposed in Pierce and Siebrits [263] as another modification of the trapezoidal rule, with some calculations performed at a full time step and the reaming ones at half-time steps. Another approach presented in Marrero and Dominguez [224] is based on a constant velocity prediction algorithm and introduces a combination of the integral representation of displacements over several consecutive time steps. The temporal distribution of tractions is constant within each time step, with the expected drawback of diminished accuracy. Recently, a new stabilization method was presented by Soares and Mansur [320], labeled as the alpha-delta method, whereby a stabilization parameter is used to modify the most recent time convolution operations by replacing current values by weighted averages. Then, matrix interpolations are used to approximate the remote (in time) convolution contributions. This averaging procedure was applied to 2D problems in elastodynamics, although in principle it can be used for 3D cases.

Stabilizing procedures are often accompanied by reduction in accuracy, resulting from added artificial damping and natural period elongation. Moreover, these techniques are somewhat cumbersome and with different levels of complexity when it comes implementation in TD-BEM computer codes (Dominguez [93]). In recent work by Panagiotopoulos and Manolis [248], the velocity reciprocal theorem is introduced as an alternative approach and tested for 1D axial wave propagation, where superior performance regarding stability was observed. Velocity-based (instead of displacement-based) identities have also been used before in Carrer and Mansur [52], but as a consequence of time differentiation of the displacement's integral representation. More specifically, the velocity identity is utilized of 2D elastodynamics so as to provide further equations and in combination