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General formulation and solution procedure for harmonic response of rigid foundation on isotropic as well as anisotropic multilayered half-space

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ABSTRACT

A general formulation and solution procedure are proposed for harmonic response of rigid foundation on multilayered half-space. It is suitable for isotropic as well as anisotropic soil medium. The wave motion equation is formulated in frequency wave-number domain in the state space. A hybrid approach is proposed for its solution, where the precise integration algorithm (PIA) is employed to carry out the integration. Very high accuracy can be achieved. The mixed variable form of wave motion equation enables the assembly of layers simple and convenient. The surface Green's function is regarded as rigorous, because it is free from approximations and discretization errors. The algorithm is unconditionally stable. The numerical implementation is based on algebraic matrix operation. Numerical examples of vibration of rigid foundation validate the efficiency and accuracy of the proposed approach.

1. Introduction

Dynamic soil–structure interaction (SSI) is a subject of considerable scientific and practical importance. The harmonic response of the foundation constitutes one of the key elements in the formulation of SSI problem. Typical applications of SSI concern a great number of problems in many engineering fields. In this paper, a general formulation for harmonic response of rigid foundation on isotropic as well as anisotropic multi-layered half-space is developed, and a hybrid approach is proposed for its solution.

Research on SSI problems progressed rapidly in the second half of 20th century stimulated by the needs of the nuclear power, hydroelectric power and offshore industries, by the development of powerful computers and advanced numerical simulation method, such as the finite element method (FEM), the boundary element method (BEM) and the thin-layer method (TLM), etc.

In 1936, the publication of Reissner [1] attempted the first engineering application of the theory of wave propagation in an elastic continuum. This marked the beginning of the modern dynamics of SSI analysis. Though the solution of Reissner was only an approximate one, as the distribution of contact stresses between the foundation and the underlying soil being assumed in advance, he offered a major contribution by clarifying the existence of radiation damping – a phenomenon previously unsuspected but today clearly understood. The outgoing waves carry away part of the energy, transmitted by the foundation on the infinite soil, which plays an important role in the dynamic response of superstructure.

More representative rigorous solution appeared in 60s till 80s of the last century. The vibration of soil-structure system was analyzed as a mixed boundary value problem by employing integral equation's approach, or by the solution based on the point Green's functions [2–6]. The analytical method based on the theory of elastic wave motion ensures strictness and stability of the solution and helps better understanding of the essence of the problem; however, the numerical implementation becomes complicated and difficult.

The discrete numerical method emerged as a promising alternative. It extends the range of the solution, which is not easily amenable to analytical approach. It raised the possibility to get access to the problems concerning the flexibility of the foundation, depth of embedment, multi-layered strata, etc.

FEM is simple and suitable in handling arbitrary material properties and complicated geometric shapes of the foundation and the soil. The major difficulty of FEM arises from proper modeling of the unbounded soil domain. Special wave absorbing boundaries must be imposed at the truncated surfaces to account for the radiation energy into the outer soil domain. Truly three dimensional FEM analysis is costly.

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BEM is well suited to model infinite medium as the radiation condition is satisfied automatically [7]. It is computational efficient because only the boundary needs to be discretized resulting in a reduction of the spatial dimension by one. However, for BEM analysis, the fundamental solution is required. Especially for multi-layered strata the BEM formulation is based on full space fundamental solution, a discretization of the soil–foundation interface and the surrounding free surface as well and the interfaces between the soil layers is necessary.

The TLM has been regarded by many researchers as an efficient and versatile technique dealing with the problem of wave propagation in layered soils. It was incepted by Lysmer and Waas [8,9] and further developed by Kausel [10,11] and many other researchers. In the meantime Tajimi et al. published their papers on TLM in Japanese journals [12,13]. TLM is semi-analytical in the sense that it combines a FE discretization in the direction of layering with an analytical solution in the direction of wave propagation in the frequency-wave number domain. The presence of underlying half-space is considered using a paraxial approximation by adding an appropriately thick buffer layer. All the physical layers and the buffer layer are divided into thin sub-layers. The displacements within the sub-layer are assumed to vary as a prescribed distribution. The selected thickness of the sublayer affects the stability and accuracy of the results.

Despite significant progress has been achieved in the investigation of SSI analyses in the past decades, and a survey of the published literature has shown that most studies were concentrated on idealized soil profiles and a homogeneous half-space has been tacitly assumed in most of the actually available programs. On the contrary, the SSI problems which we have to face in practice necessitate tackling it under more complicated soil situations. Even more, soils and rocks in nature invariably exhibit some degree of anisotropy in their response to static or dynamic stresses. To study SSI in anisotropic multi-layered soil media is a real challenge, few works can be found in the literature. Gazetas [14] proposed a semi-analytical approach to strip foundation on layered soil strata. The wave motion equation was uncoupled into pseudo-distortional and pseudo-dilational wave potentials based on transformation, and then analytical expressions can be obtained for displacements and stresses in each layer. Kausel [15], Barbosa and Kausel [16] generalized TLM to cross-anisotropic 3D stratified media. The displacement field within each layer is expressed by means of a prescribed distribution. No concrete numerical example dealing with anisotropic soil strata was provided. Zhong et al. [17] and Gao et al. [18] developed a precise integration algorithm (PIA) to solve the generalized plane wave propagation problem in anisotropic stratified soil. Khojasteh et al. [19] achieved three-dimensional dynamic Green's functions for a multi-layered transversely isotropic half-space. The authors of this paper [20] presented an approach for dynamic impedance of strip foundation on anisotropic layered half-space based on PIA.

In this paper, a hybrid approach is presented for the solution of wave propagation in multilayered half-space in frequency-wavenumber domain. The media can be of either isotropic or anisotropic. Analytical expressions are obtained for displacements and stresses in each layer, which is free from approximation and discretization errors. The computation is unconditionally stable. The precise integration algorithm ensures highly accurate results. As a numerical example vibration of foundation on multi-layered half-space is studied, the interface between the foundation and the soil is discretized into a number of uniformly spaced nodal points or subdisk-elements, which makes the method quite feasible for handling arbitrary shaped foundations. The solution of the surface Green's function for the multi-layered half-space is regarded as rigorous, that is free from approximations and discretization errors. Numerical examples validate effectiveness of the proposed approach. Part of the contents has been presented at a conference held in Aachen, Germany [21]. This paper is an improved version of the work.

2. Governing equation in frequency-wavenumber domain

A multi-layered soil strata including l layer as shown in Fig. 1 is considered. For a typical layer of general anisotropic medium, the stress–strain relationship in Cartesian coordinates is expressed as follows:

$$\boldsymbol{\sigma} = D\boldsymbol{\varepsilon} \tag{1}$$

where

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{yz} & \tau_{xz} & \tau_{xy} \end{bmatrix}^T$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{yz} & \gamma_{xz} & \gamma_{xy} \end{bmatrix}^T$$

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ & & d_{33} & d_{34} & d_{35} & d_{36} \\ & & & d_{44} & d_{45} & d_{46} \\ & \text{(sym)} & & & d_{55} & d_{56} \\ & & & & & d_{66} \end{bmatrix} \tag{2}$$

The wave motion equation takes the form

$$\mathbf{D}_{xx}\frac{\partial^2 \mathbf{q}}{\partial x^2} + \mathbf{D}_{yy}\frac{\partial^2 \mathbf{q}}{\partial y^2} + \mathbf{D}_{zz}\frac{\partial^2 \mathbf{q}}{\partial z^2} + \left(\mathbf{D}_{xy} + \mathbf{D}_{yx}\right)\frac{\partial^2 \mathbf{q}}{\partial x \partial y} + \left(\mathbf{D}_{yz} + \mathbf{D}_{zy}\right)\frac{\partial^2 \mathbf{q}}{\partial y \partial z}$$
$$+ (\mathbf{D}_{xz} + \mathbf{D}_{zx})\frac{\partial^2 \mathbf{q}}{\partial x \partial z} = \rho \ddot{\mathbf{q}} \tag{3}$$

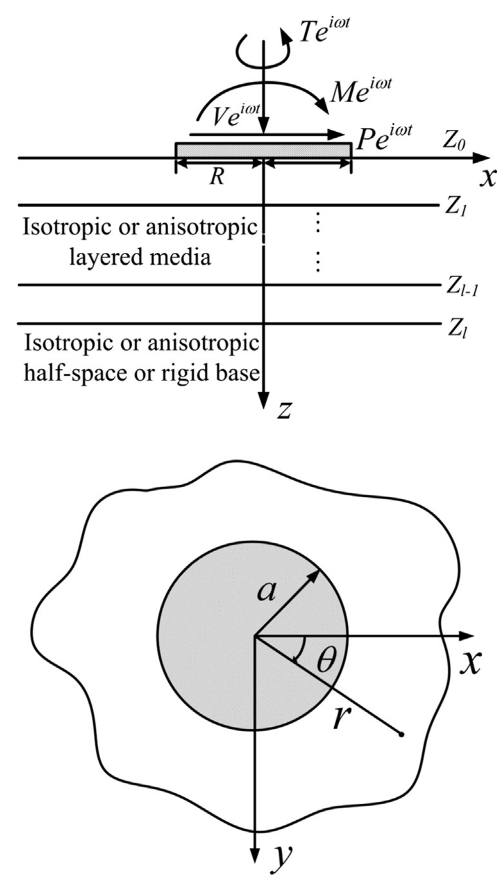

Fig. 1. Model of multilayered half-pace.