Soil Dynamics and Earthquake Engineering 70 (2015) 75–79

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Technical Note

Inelastic seismic spectra including a damage criterion: A stochastic approach

Rita Greco [a,*,1], Giuseppe Carlo Marano [b,1]

[a] DICATECH, Department of Civil Engineering, Environmental, Territory, Building and Chemical, Technical University of Bari, via Orabona 4, 70125 Bari, Italy
[b] DICAR, Department of Civil Engineering and Architecture, Technical University of Bari, via Orabona 4, 70125 Bari, Italy

ARTICLE INFO

Article history:
Received 26 November 2014
Accepted 30 November 2014
Available online 1 January 2015

Keywords:
Damage-based seismic spectra
Park–Ang damage model
Stochastic process
Peak theory

ABSTRACT

In this paper, a stochastic approach for obtaining damage-based inelastic seismic spectra is proposed. The Park and Ang damage model, which includes displacement ductility and hysteretic energy, is adopted to take into account the cumulative damage phenomenon in structural systems under strong ground motions. Differently from previous studies in this field, damage-based seismic spectra are obtained by means of peak theory of stochastic processes. The following stochastic inelastic seismic spectra are constructed and then analyzed: damage-based displacement and acceleration inelastic spectra, damage-based response modification factor spectra, damage-based yield strength demand spectra and damage-based inelastic displacement ratio spectra.

1. Introduction

The requirement of ductile behavior is the conceptual base in modern seismic design, that means with a sufficient ductility a certain degree of damage can be tolerated during moderate to severe earthquakes. In current design Codes, inelastic ductile behavior is taken into account through the *response modification factor* RMF that allows structures to be designed for lateral forces smaller than those required to remain elastic during severe earthquakes. Inelastic design spectra are derived from elastic ones scaling-down of RMF (q and R for Eurocode8 [1] and US FEMA, respectively, [2]). For structures that exhibit inelastic behavior, the *strength reduction factor* SRF should be introduced. In literature the SRF is known as ductility-based SRF R_μ. As widely known, every real structure has a limited ductility. In a non-linear system subjected to a given earthquake, if the yield strength is very small it will have large post-elastic displacement exceeding the available ductility. To limit the displacement response within the available ductility the yield strength must be increased. Denoting the required minimum yield strength of a system with available ductility μ by $F_{y,\mu}$ and the maximum force of a linear elastic system by F_e, the ductility-based SRF may be defined by the ratio of the elastic strength demand F_e to the yield strength demand $F_{y,\mu}$, by which the required ductility of the system would be μ for a prescribed level of ground motion. The R_μ factor has been the subject of numerous investigations ([3–6]). The design spectra built using the R_μ factor are known as ductility-based inelastic demand spectra and are conventionally adopted in strength based seismic design. The methodologies developed for obtaining ductility-based seismic spectra suffer however from a conceptual limitation; indeed the inelastic spectra constructed in this way are significant only on the maximum demand of ductility and they do not include any influence of number of response cycles, yield excursions, stiffness and strength degradation and damage potential to structures. It is evident that, especially under long duration seismic actions, yielding structures experience an increased number of reversals of inelastic deformations and the accumulation of damage may significantly influence structure performance

The introduction of damage phenomena in seismic spectra is not new and various proposals have been developed for cyclic demand spectra that can modify classic spectra for a more accurate definition of the seismic load ([4,7–16]). However, in most of these studies inelastic spectra including damage criteria are constructed considering a single earthquake record or alternatively are developed on the basis of repeated statistical analyses of non-linear system response to a large number of ground motion records.

In this paper, a stochastic approach for obtaining damage-based inelastic spectra is proposed. The Park and Ang damage model, which includes displacement ductility and hysteretic energy, is used to take into account the cumulative damage phenomenon in structural systems under strong ground motions. The non-linear Bouc–Wen model, characterized by a smooth hysteresis and a post-yielding stiffness, is adopted to develop inelastic stochastic

* Corresponding author.
E-mail addresses: r.greco@poliba.it (R. Greco), g.marano@poliba.it (G.C. Marano).
[1] Tel./fax: +39 0805963875.

http://dx.doi.org/10.1016/j.soildyn.2014.11.012
0267-7261/

seismic spectra. The main advantage of stochastic approach consists in the possibility to typify seismic motion by few parameters, containing the most important characteristics, such as the frequency content, the peak acceleration, the energy content and the strong motion duration, i.e. all parameters that affect structural response under seismic actions.

2. Structural model and stochastic seismic analysis

2.1. Bouc–Wen model

Various kinds of hysteresis models have been formulated to represent the response of steel and reinforced concrete members under cyclic actions. A very diffused model is the Bouc–Wen one [17,18]. In this model the restoring force $Q\,(x,\dot{x},z)$ can be divided into two parts. The first $c\,\dot{x}$ is due to the viscous–elastic contribution, while the second $F(x,\dot{x},z)$ is due to the hysteretic one:

$$Q\,(x,\dot{x},z) \;=\; c\,\dot{x} \;+\; \alpha\,k\,x \;+\; (1-\alpha)k\,z = c\,\dot{x} + F(x,\dot{x},z) \tag{1}$$

In Eq. (1) x is the oscillator displacement, c and k are the system damping and the elastic stiffness, z is an internal variable related to the hysteretic behavior, satisfying the following differential equation:

$$\dot{z}(t) = \dot{x}(t)[\lambda - |z(t)|^n\,(\beta + \gamma \cdot \text{sgn}\,\{z(t)\}\,\cdot \text{sgn}\,\{\dot{x}(t)\})] \tag{2}$$

The five parameters $\beta, \gamma, \eta, \alpha$ and λ are the shape factors of the hysteretic cycle and are well described by Cunha [19]. The Bouc–Wen model has been and it is still now commonly utilized in the field of seismic engineering for the analysis of conventional reinforced concrete and steel structures, and for the study of passive seismic protection systems, as base isolators [20] and dissipative devices.

For a single degree of freedom system with a mass m, subjected to a seismic action $\ddot{x}_g(t)$ and characterized by a hysteretic constitutive law described by means of the Bouc–Wen model, the motion equation is:

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha k x(t) + (1-\alpha)kz(t) = -m\ddot{x}_g(t) \tag{3}$$

By introducing $\omega = \sqrt{\frac{k}{m}}$ and $\xi = \frac{c}{2\sqrt{km}}$, Eq. (3) becomes:

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \alpha\omega^2 x(t) + (1-\alpha)z(t)\omega^2 = -\ddot{x}_g(t) \tag{4}$$

2.2. Energy equation and damage index

For the Bouc–Wen system the power balance equation is obtained by multiplying each term of the motion Eq. (3) for the velocity term$\dot{x}$:

$$m\ddot{x}\dot{x} + c\dot{x}\dot{x} + \alpha k x\dot{x} + k(1-\alpha)z\dot{x} = -m\ddot{x}_g\dot{x} \tag{5}$$

The power balance expressed through Eq. (5) is called *relative power equation* because it supplies the *relative energy equation* obtained by integrating Eq. (5) with respect to the time t:

$$\int_0^t m\ddot{x}\dot{x}dt + \int_0^t c\dot{x}\dot{x}dt + \int_0^t \alpha k x\dot{x}dt + \int_0^t k(1-\alpha)z\dot{x}dt = -\int_0^t m\ddot{x}_g\dot{x}dt \tag{6}$$

Hence, the *relative energy balance* at the time t can be written as:

$$E_k(t) + E_d(t) + E_s(t) + E_h t) = E_i(t) \tag{7}$$

where different terms are, respectively, the kinetic energy, the energy dissipated by damping, the elastic energy, the hysteretic dissipated energy and the input energy at the time t:

In literature, the most frequently used dimensional damage variables are the displacement ductility μ_x and the dissipated energy E_h:

$$\mu_x = \frac{x_{max}}{x_y} \text{ and } E_h = (1-\alpha)k\int_0^t z\dot{x}(\tau)d\tau \tag{8}$$

The well-known Park and Ang damage index [21], based on extensive experimental tests on concrete structures, considers both the ductility and the dissipated energy. This is expressed by:

$$D_{PA} = \frac{x_{max}}{x_{u,mon}} + \beta\frac{E_h}{F_y x_{u,mon}} = \frac{\mu_x + \beta(\mu_e - 1)}{\mu_{u,mon}} \tag{9}$$

In Eq. (9) μ_e is the *hysteretic ductility*:

$$\mu_e = \frac{E_h}{F_y x_y} + 1 = \frac{e_h}{\omega^2 x^2{}_y} + 1 \tag{10}$$

and $\mu_{u,mon}$ is the admissible ductility under a monotonic load:

$$\mu_{u,mon} = \frac{x_{u,mon}}{x_y} \tag{11}$$

being $x_{u,mon}$ the ultimate displacement obtained under a monotonic load analysis. β is a constant representing the rate of damage accumulation through hysteretic energy due to cyclic loading.

2.3. Stochastic seismic analysis

A stochastic seismic analysis is carried out to evaluate seismic response quantities needed to construct stochastic damage-based seismic spectra. For this purpose, the non-stationary stochastic Kanai–Tajimi model [22] is adopted to capture the inner random nature of earthquakes. The motion equations for the single degree of freedom system modeled by means of the non-linear hysteretic Bou–Wen model, subjected to the uniformly modulated Kanai–Tajimi stochastic process, are:

$$\begin{cases} \ddot{x}(t) + 2\xi\omega\dot{x}(t) + \alpha\omega^2 x(t) + (1-\alpha)z(t)\omega^2 = -\ddot{x}_g(t) = \ddot{x}_f(t) + \varphi(t)w(t) \\ \ddot{x}_f(t) + 2\xi_f\omega_f\dot{x}_f(t) + \omega_f^2 x_f = -\varphi(t)w(t) \\ \dot{z}(t) = \dot{x}(t)\left[\lambda - |z(t)|^\eta\left(\beta + \gamma \cdot sign\{z(t)\} \cdot sign\{\dot{x}(t)\}\right)\right] \end{cases} \tag{12}$$

In Eq. (12) $w(t)$ is a stationary Gaussian zero-mean white noise stochastic process, and $\varphi(t)$ is the temporal modulation function. Moreover, x_f is the response of the Kanai–Tajimi filter, having a frequency ω_f and a damping coefficientξ_f; $\ddot{x}_g$ is the excitation process at the base of the structure with constant Power Spectral Density function S_0. The Jennings [23] modulation function is adopted in this study:

$$\varphi(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2 & t < t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-\vartheta(t-t_2)} & t > t_2 \end{cases} \tag{13}$$

In this paper, the equivalent stochastic linearization method [24] and Liapunov equation [25] are adopted to obtain stochastic seismic response. Moreover, to develop damage-based stochastic spectra, D_{PA} should also be evaluated in stochastic terms. The authors in a previous study [26] furnish a detailed description of the procedure.

3. Construction of damage-based inelastic seismic spectra

3.1. Peak theory of stochastic processes

The response spectrum is given by the plot of the maximum response of a single degree of freedom system to the recorded earthquake versus its natural period. In stochastic meaning, in an analog way, the response spectrum is the plot of the maximum