Soil Dynamics and Earthquake Engineering 60 (2014) 51–68

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Application of perfectly matched layers in the transient analysis of dam–reservoir systems

Adib Khazaee *, Vahid Lotfi

Department of Civil and Environmental Engineering, Amirkabir University of Technology, 424 Hafez, Ave., Tehran, Iran

ARTICLE INFO

Article history:
Received 11 February 2013
Received in revised form
19 December 2013
Accepted 7 January 2014

Keywords:
Wave propagation
Semi-infinite domains
Perfectly matched layers
Dam–reservoir systems
Time domain analysis

ABSTRACT

The transient analysis of dam–reservoir systems by employing perfectly matched layers has been investigated. In previous studies, boundary conditions of the PML region in the reservoir have been neglected. In this paper, they are incorporated completely in the formulation. Moreover, a technique is introduced to involve the effect of incident waves caused by vertical ground motions at the reservoir bottom in the analysis. Performing several numerical experiments indicates that applying boundary conditions of the PML domain and utilizing the proposed method for vertical excitation cases reduce the computational cost significantly and make the PML method a very efficient approach for the transient analysis of dam–reservoir systems.

1. Introduction

Physicists and engineers have been seeking reliable approaches to analyze processes involving wave motions for decades. Particularly, one encounters acoustic waves propagating in a semi-infinite medium in the transient analysis of dam–reservoir systems (Fig. 1). In numerical models of such systems, acoustic finite elements are utilized to discretize the near field part of the reservoir adjacent to the dam body (with irregular geometry). Meanwhile, researchers have suggested many approaches such as hyper-elements [1], rational boundary conditions [2,3], Dirichlet to Neuman mappings [4], the boundary element method [5], the scaled boundary element method [6] and high order non-reflecting boundary conditions [7] to take into account the propagation of acoustic waves towards infinity in the analysis. Nevertheless, finding methods for applying the radiation condition as thoroughly and efficiently as possible is still the purpose of many investigations. The present study is focused on utilizing perfectly matched layers in the transient dynamic analysis of dam–reservoir systems.

A perfectly matched layer is an absorbing layer which can absorb propagating waves perfectly if it is defined properly. Berenger (1994) introduced perfectly matched layers for solving unbounded electromagnetic problems with the finite-difference time-domain method [8]. Hasting seems to be the first researcher who used perfectly matched layers in problems including elastic waves [9]. He split potential functions corresponding to primary and secondary waves and utilized a finite-difference time-domain (FDTD) approach to solve the resultant equations in 2D domains. Chew et al. introduced a change of variables to transform Maxwell's equations in PML media into ordinary-looking Maxwell's equations in a complex coordinate system. They indicated that many existing closed-form solutions can be easily mapped into solutions in these complex coordinate systems [10]. Chew and Liu employed complex coordinates to define perfectly matched layers and showed that the resultant medium could absorb propagating waves [11]. Issac Harari et al. presented a finite element formulation to use PML in time harmonic analysis of acoustic waves in exterior domains [12]. Collino and Tsogka indicated how to establish a PML model using the split-field approach for a general hyperbolic system. They implemented their theory to the linear elastodynamic problem in an anisotropic medium [13]. Zeng et al. extended the PML to truncate unbounded poroelastic media for numerical solutions using a finite-difference method. They adopted the method of complex coordinates to formulate the PML for poroelastic media [14]. Zheng and Huang developed anisotropic PML for elastic waves in Cartesian, cylindrical and spherical coordinates. Their formulation avoided field splitting and could be used in the FEM directly, and in the FDTD method too [15]. Becache et al. investigated well-posedness and stability of using perfectly matched layers for anisotropic elastic waves from a theoretical point of view [16]. Basu and Chopra defined perfectly matched layers by employing complex coordinates to solve time harmonic elastodynamic equations by finite

* Corresponding author. Tel.: +98 912 7178738.
E-mail addresses: adibkhazaee@aut.ac.ir, adib_kh7@yahoo.com (A. Khazaee), vahlotfi@aut.ac.ir (V. Lotfi).

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http://dx.doi.org/10.1016/j.soildyn.2014.01.005

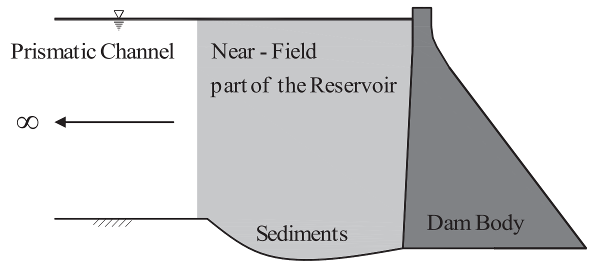

Fig. 1. A typical dam–reservoir system containing the dam body, the irregular part of the reservoir, the semi-infinite channel and sediments at the reservoir bottom.

element implementation [17]. Furthermore, they transformed the frequency domain equations into time domain and presented an approach to solve the resultant equations [18,19]. Katsibas and Antonopoulos implemented a FDTD-PML technique to solve stress–velocity acoustic equations. They derived general PML equations governing both lossless and lossy media. Moreover, they used the stretched coordinates idea to introduce further dissipation into the PML area [20]. Rylander and Jin developed a formulation for PML to solve Maxwell's equations by finite element method. They transformed frequency dependent Maxwell's equations into time domain and introduced a special time stepping scheme to solve the resultant equations [21]. Appelo and Kreiss utilized the formulation of a modal PML to the equations of linear elasticity. They indicated that their PML model has better stability properties than previous split-field models [22]. Issac Harari et al. conducted a parametric study on PML used in the time harmonic analysis of elastodynamics in an unbounded region by the finite element method and presented some guidelines for choosing PML parameters [23]. Shuo Ma and Pengcheng Liu presented an easy implementation of perfectly matched layers (PML) in the explicit finite element method by using the one-point integration scheme [24]. Qin et al. introduced auxiliary variables to divide the PML wave equation in the frequency domain into two parts: normal terms and attenuated terms. Using the auxiliary variables, they avoided convolution operations in equations after transforming them into time domain and utilized the finite difference method to propose a novel numerical implementation approach for PML absorbing boundary conditions with simple calculation equations, small memory requirements, and easy programming [25]. Liu et al. utilized the Crank–Nicolson scheme together with several algorithms to calculate the first-order spatial derivatives of the SH wave equations. Furthermore, they investigated how the absorbing boundary width and the algorithms affect the PML results of a homogeneous isotropic medium and a multi-layer medium with a cave [26]. Seungil Kim and Joseph E. Pasciak developed a Cartesian perfectly matched layer for solving Helmholtz equation on an unbounded domain in 2D space [27]. Giovanni Lancioni compared the performance of the PML approach and high order non-reflecting boundary conditions in a one dimensional dispersive problem and expressed their merits and drawbacks [28].

In the analysis of a dam–reservoir system by perfectly matched layers, the bounded part of the reservoir is discretized by common finite elements and a PML region is modeled to absorb waves propagating towards infinity (Fig. 2).

In the analysis of a dam–reservoir system by perfectly matched layers, the bounded part of the reservoir is discretized by common finite elements and a PML region is modeled to absorb waves propagating towards infinity (Fig. 2). In a previous study where Basu [29] employed PML in the dam–reservoir systems, boundary conditions of the PML domain have been neglected which results in larger domain sizes. Basu concluded that when wave absorptions at the reservoir bottom are to be considered the length of the reservoir should be at least twice as long as its height to obtain results with proper accuracy. He also implied that when the excitation has a vertical component the reservoir should be at least 6 times as long as its height [29]. In such cases, a relatively large part of the reservoir adjacent to the dam body has to be modeled using conventional acoustic elements to include the effect of wave absorptions or the vertical excitation at the reservoir bottom in the analysis. Furthermore, PML media only absorb waves generated in the bounded domain, hence, a particular approach should be employed to incorporate incident waves generated in the exterior domain due to vertical ground motions at the reservoir bottom. In a previous study, we investigated the time harmonic analysis of dam–reservoir systems by using perfectly matched layers and demonstrated that applying proper boundary conditions for the PML area results in considerably smaller domain sizes [30]. Here, perfectly matched layers introduced by Basu [29] are adopted in the time domain analysis of dam–reservoir systems while the bounded part of the reservoir adjacent to the dam body is reduced by applying boundary conditions of the PML area. The contribution of this study is threefold:

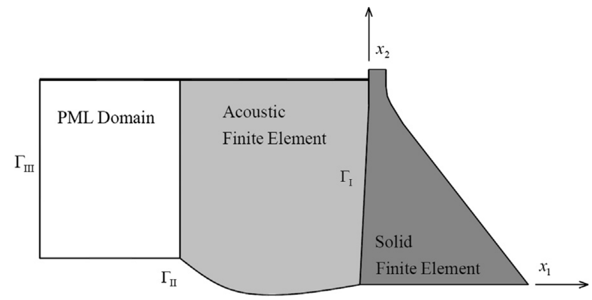

Fig. 2. Dam–reservoir FE model, the dam body discretized by solid finite elements, the near-field part of the reservoir discretized by acoustic finite elements, the far-field part of the reservoir is discretized by PML finite elements.

- Considering the effect of applying Sommerfeld boundary condition at the truncation boundary of the PML area.
- Applying a boundary condition at the bottom of the PML area to include wave absorptions at the reservoir bottom.
- Proposing a method for the transient analysis of a semi-infinite reservoir involving vertical ground motions at the reservoir bottom.

2. Formulation of dam–reservoir systems in time domain

Combining equations governing the solid medium of the dam body and water in the reservoir results in the formulation of dam–reservoir systems. Employing finite element method, one can write the equation of motion of the dam body as follows [31]:

$$\mathbf{M}\ddot{\mathbf{r}}+\mathbf{C}\dot{\mathbf{r}}+\mathbf{K}\mathbf{r}=-\mathbf{M}\mathbf{J}\mathbf{a}_g+\mathbf{B}^T\mathbf{P} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the dam's mass, damping and stiffness matrices, $\mathbf{a}_g$ is the ground acceleration vector and $\mathbf{r}$ is the vector of nodal relative displacements. $\mathbf{J}$ is a matrix which applies the ground acceleration to nodes of the model and the interaction matrix $\mathbf{B}$ transforms nodal hydrodynamic pressures $\mathbf{P}$ into nodal forces on the upstream face of the dam.

If water in the reservoir is assumed to be inviscid, linearly compressible with small irrotational movements, hydrodynamic pressures can be determined through the following differential equation and proper boundary conditions:

$$\nabla^2 p-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=0 \text{ in the reservoir} \tag{2}$$

$$p=0 \quad \text{at the water surface} \tag{3a}$$

$$\nabla p\cdot\mathbf{n}_d=-\rho\,\mathbf{n}_d\cdot\ddot{\mathbf{u}} \quad \text{on the dam–reservoir interface, } \Gamma_I \tag{3b}$$