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Natural vibration frequency and damping of slender structures founded on monopiles

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ABSTRACT

Offshore wind turbine (OWT) is a typical example of a slender engineering structure founded on large diameter rigid piles (monopiles). The natural vibration characteristics of these structures are of primary interest since the dominant loading conditions are dynamic. A rigorous analytical solution of the modified SSI eigenfrequency and damping is presented, which accounts for the cross coupling stiffness and damping terms of the soil–pile system and is applicable but not restrictive to OWTs. A parametric study was performed to illustrate the sensitivity of the eigenfrequency and damping on the foundation properties, the latter being expressed using the notion of dimensionless parameters (slenderness ratio and flexibility factor). The application of the approximate solution that disregards the off diagonal terms of the dynamic impedance matrix was found to overestimate the eigenfrequency and underestimate the damping. The modified SSI eigenfrequency and damping was mostly affected by the soil–pile properties, when the structural eigenfrequency was set between the first and second eigenfrequency of the soil layer. Caution is suggested when selecting one of the popular design approaches for OWTs, since the dynamic SSI effects may drive even a conservative design to restrictive frequency ranges, nonetheless along with advantageous – from a designers perspective – increased damping.

1. Introduction

Piles are widely used as a foundation solution for several engineering structures, including tall slender structures, where extreme loading conditions or soft soil conditions may lead to the design and construction of large diameter monopiles due to deformation limitations in the superstructure. Hence stiff, large diameter single piles comprising the foundation system of slender structures are met at energy transmission towers, offshore wind turbines (OWTs), or tall bridge piers. Dynamic response should be considered in the design of the aforementioned structures, since they are subjected to wind and earthquake loading. Essential to this end is the determination of the natural vibration characteristics, i.e. eigenfrequencies and damping, of the coupled system accounting for the effect of the interaction of the pile with the soil. In the case of offshore wind turbines this issue has been considered of outmost importance and is portrayed in the relevant design standards [1] and certification guidelines [2]. According to the latter the eigenfrequency of the tower should be either greater or smaller than the blade passing frequency and the rotor frequency in normal operating mode. Even though such a restriction is not clearly reported in DNV standard [1] the calculation of the natural frequencies of the tower is crucial for the prediction of the wave, ice, and wind load effects.

Furthermore in the current practice three design philosophies have prevailed, being established after consideration of different combinations of tower′s stiffness and foundation′s stiffness, the so-called soft–soft, soft–stiff and stiff–stiff design [3,4]. In the first design approach the tower and the foundation are designed so that the eigenfrequency is less than the rotor frequency, it is thus reduced to excessively low values, where the cost reduction is considerable but wave fatigue may be problematic [5]. In the third one the eigenfrequency is larger than the blade passing frequency, where the wind induced fatigue is an additional issue to be dealt with, except from the cost increase. In the second one the eigenfrequency falls between the rotor frequency and the blade passing frequency, and attains the advantages of (a) minimising the uncertainty induced by the soil conditions, (b) reducing the production cost, and (c) reducing the dynamic wave loading [5].

The significance of the soil properties and the soil–pile-tower interaction, in the estimation of the eigenfrequency of an OWT, has been underlined in both certification and design guidelines [2,6]. Moreover recent research has been directed towards the development of analytical and experimental methods allowing for the calculation of the eigenfrequency of the wind turbine, while incorporating also the influence of the foundation stiffness [7–10]. On the other hand the influence of the adopted foundation modelling in the eigenfrequency of OWT has been also examined [11–13]. The prevailing modelling approach relies on the former analysis of the soil–pile interaction and the estimation of the corresponding stiffness coefficients at the mudline. The dynamic

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Notation

Latin upper case

D	diameter of OWT
E_s	soil modulus of elasticity
E_p	effective Young modulus of pile by Randolph [29]
G	soil shear modulus
G^*	shear modulus parameter by Randolph [29]
H_s	height of structure
H	height of pile and soil layer
I	moment of inertia of pile
K_r	pile flexibility factor
$\tilde{K}_{su}$	complex valued impedance – force for unit displacement
$\tilde{K}_{mu}$	complex valued impedance – moment for unit displacement
$\tilde{K}_{s\vartheta}$	complex valued impedance– force for unit rotation
$\tilde{K}_{m\vartheta}$	complex valued impedance – moment for unit rotation
K_{su}	dynamic stiffness coefficient – real part of $\tilde{K}_{su}$
K_{mu}	dynamic stiffness coefficient – real part of $\tilde{K}_{mu}$
$K_{s\vartheta}$	dynamic stiffness coefficient – real part of $\tilde{K}_{s\vartheta}$
$K_{m\vartheta}$	dynamic stiffness coefficient – real part of $\tilde{K}_{m\vartheta}$
$K^0{}_{su}$	static stiffness coefficient – force for unit displacement
$K^0{}_{mu}$	static stiffness coefficient – moment for unit displacement
$K^0{}_{s\vartheta}$	static stiffness coefficient – force for unit rotation
$K^0{}_{m\vartheta}$	static stiffness coefficient – moment for unit rotation
M_r	reaction moment at the pile head
P_h	horizontal reaction force at the pile head
T	eigenperiod of structure
V_s	soil shear wave velocity
Y	relative pile stiffness

Latin lower case

d	diameter of pile
$\tilde{f}$	modified SSI eigenfrequency
f_s	eigenfrequency of structure
k_s	stiffness of structure
m_s	mass of structure
u_p	translational degree of freedom at the pile head
u_g	amplitude of harmonic applied displacement
$\tilde{u}_g$	modified SSI amplitude of ground displacement

Greek

α_0	dimensionless eigenfrequency of soil layer
$\tilde{\zeta}$	modified SSI relative damping ratio
ζ_s	damping ratio of structure
ζ_{su}	damping coefficient – force for unit displacement
ζ_{mu}	damping coefficient – moment for unit displacement
$\zeta_{s\vartheta}$	damping coefficient – force for unit rotation
$\zeta_{m\vartheta}$	damping coefficient – moment for unit rotation
ϑ	rotational degree of freedom at the pile head
λ	dimensionless eigenfrequency of freestanding pile
ν	soil's Poisson's ratio
ξ	hysteretic soil damping ratio
ρ	density of soil
$\tilde{\omega}$	modified SSI circular eigenfrequency
ω_s	circular eigenfrequency of the fixed base structure
ω	circular frequency of harmonic applied displacement
ω_{app}	approximate eigenfrequency disregarding the cross coupling impedance coefficients

impedances provide the basis for the calibration of the corresponding model parameters, leading thus to a two-step procedure, which had emerged from a three step procedure developed in the early 1970s for the dynamic soil–structure-interaction (SSI) analysis [14].

Extensive research effort has been placed on the estimation of the dynamic impedances in horizontal vibration of single piles. The analytical and numerical studies reported in the literature can be broadly categorised as (a) rigorous elastic continuum analytical solutions [15–18], (b) elastic Winkler type analytical solutions, where the soil is considered as a horizontally layered medium [19–23], (c) numerical lumped mass and discrete element models [24–27] and (d) numerical continuum finite element solutions [28–33]. Analytical solutions [15] even though limited to elastic soil response, have been so far useful in acquiring a better understanding of the main phenomena of soil–pile interaction and have been shown to compare well with finite element results [28,29]. Nevertheless, comparing the theoretical basis of the two analytical approaches reveals that the assumed soil response in the case of Winkler type solutions, allows only for consideration of horizontally propagating waves. It is evident that the same limitation applies also when comparing lumped mass and continuum finite element models, while on the other hand incorporating nonlinear response using the latter requires sophisticated soil constitutive models and adequate modelling of the soil–pile interface. The shortcomings of the Winkler type analytical solutions especially at low frequencies were early recognised [28,20] and approximate corrections for the damping were proposed [21]. A more rigorous solution was obtained by integrating the governing equations over the thickness of the soil layer, which captures also the soil layer resonances in the variation of the dynamic stiffness with frequency [23].

The simplification of the superstructure to a single dynamic degree of freedom system (SDOF) allows for the analytical estimation of the dynamic response characteristics (see Table 1), taking into account the soil–structure interaction by the translational (K_{su}) and rotational ($K_{m\theta}$) spring coefficients and the corresponding damping ratios (ζ_{su}, and $\zeta_{m\theta}$). Simplified expressions for the eigenfrequency and damping of a SDOF on elastic half-space with massless foundation were derived by Jennings and Bielak [34], suggesting also that the off-diagonal terms could be disregarded as negligible in the case of the rigid circular disk oscillating on the top of elastic half space. Based on the same concept of a replacement oscillator but with finite foundation mass, approximate expressions for the natural frequency and damping were proposed [35]. Wolf [36] obtained the same analytical solution for the modified eigenperiod but a quite different expression for the equivalent damping (since soil damping was separated to a material and a radiation damping term), after satisfying the three governing dynamic equilibrium equations of the aforementioned system. The SSI eigenperiod was modified [37] after calibrating the results of numerical analyses, which relied on the stiffness coefficients proposed for flexible piles [32]. Recently Wolf's solution [36] was improved by accounting for the second order terms of damping (see Table 1), which had been neglected in all the above mentioned studies [38]. The latter appears to have a better correlation with the results of a numerical study where the dynamic response of three dimensional elastic continuum finite element models indicated the existence of an additional eigenperiod defined as pseudo-natural SSI eigenperiod [39].

The motivation of the current study emerges from the paradigm application of an OWT, which resembles the case of slender structures supported by single, large diameter pile foundations.