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Vertical and horizontal vibrations of a rigid disc on a multilayered transversely isotropic half-space

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ABSTRACT

A half-space containing horizontally multilayered regions of different transversely isotropic elastic materials as well as a homogeneous half-space as the lowest layer is considered such that the axes of material symmetries of different layers and the lowest half-space to be as depth-wise. A rigid circular disc rested on the free surface of the whole half-space is considered to be under a forced either vertical or horizontal vibration of constant amplitudes. Because of the involved integral transforms, the mixed boundary value problems due to mixed condition at the surface of the half-space are changed to some dual integral equations, which are reduced to Fredholm integral equations of second kind. With the help of contour integration, the governing Fredholm integral equations are numerically solved. Some numerical evaluations are given for different combinations of transversely isotropic layers to show the effect of degree of anisotropy of different layers on the response of the inhomogeneous half-space.

1. Introduction

Forced vibration of an elastic isotropic or orthotropic medium, which may be due to vibration of a surface or buried rigid plate is very interesting in mathematical theory of elasticity and its applications. Because of rigidity of the plate, the related boundary value problem is a mixed boundary value problem, which is interesting for mathematicians. The analysis is useful for understanding the mechanics of the interaction of foundations and the supporting soil under external load, and also in analysis of matrix containing flat inclusions. There are some research works in this area for both static and dynamic cases, which are useful to be mentioned [1–3]. In addition, because of the application of layered media many investigators have examined the boundary value problems in multilayered elastic materials [7]. On the other hand, the physical behavior of the soil beneath a foundation is not identically isotropic, and thus analytical treatment of the problem of a rigid disc associated with an anisotropic medium is needed. Because of the direction of gravity the transversely isotropic behavior is the most common applicable medium among the different anisotropic one for the soil. Eskandari-Ghadi [8] introduced a complete set of scalar potential functions for the elastodynamic problems related to transversely isotropic axially convex domain. With the use of these potential functions, Eskandari-Ghadi et al. [9] have studied the axisymmetric vertical forced vibration of a rigid circular disc rested on a transversely isotropic homogeneous half-space, where they have shown a singular behavior for the pressure in between the disc and the half-space.

It is the purpose of this paper to investigate forced either vertical or horizontal vibration of a rigid circular disc rested in the relaxed form on the top of a transversely isotropic multilayered half-space.

2. Boundary-value problem and its solution

A half-space containing horizontally multilayered regions of different transversely isotropic elastic materials is considered (Fig. 1). The axes of material symmetry of different layers and the lowest half-space are assumed to be depth-wise. A massless rigid disc of radius a is considered to be attached on the top of the media. As indicated in Fig. 1, a cylindrical coordinate system $\{O; x=(r,\theta,z)\}$ is attached to the domain of the problem. As a reference, the first layer, the second layer, …, and the lowest half-space are referred to as Region 1 $(0<z<h_1)$, Region 2 $(h_1<z<h_2)$, …, and Region $(n+1)(z>h_n)$, respectively. The behavior of a transversely isotropic material is completely described by five independent elastic constants $A_{11},\ A_{12},\ A_{13},\ A_{33}$ and A_{44} [10]. Usually, a dependent elasticity constant denoted as $A_{66}=(A_{11}-A_{12})/2$ is used to make the constitutive law for transversely

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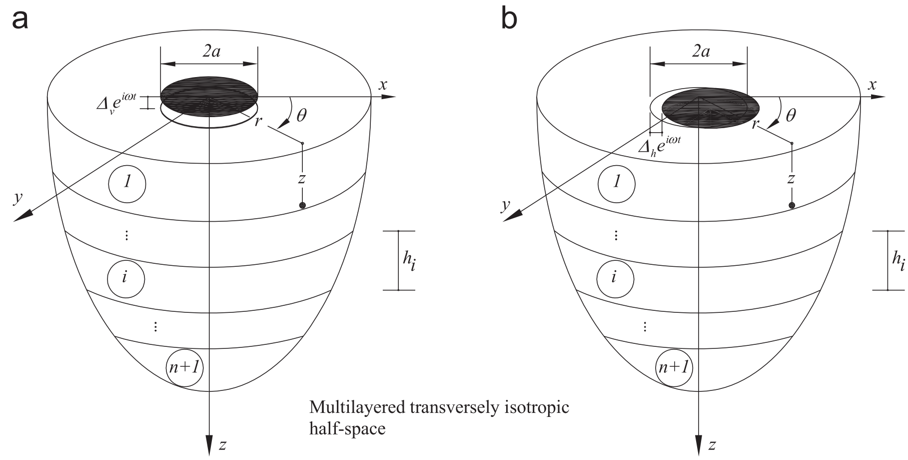

Fig. 1. Multilayered transversely isotropic half-space under a forced (a) vertical, and (b) horizontal excitation.

isotropic material to be written simpler. These elastic constants may be related to the engineering constants E, E', υ, υ', G and G' as indicated in [10]. A prescribed mono-harmonic vibration either vertical or horizontal in the form of $\Delta_v e^{i\omega t}$ or $\Delta_h e^{i\omega t}$ is considered for the disc, where Δ_v and Δ_h are the amplitudes of the vertical and horizontal movements and ω is the circular frequency of the motion. Writing the time-harmonic equations of motion in terms of displacements in each region as in [11] and utilizing the scalar potential functions F and χ, the solutions for the potential functions in the jth layer are (see [11,12]):

$$\begin{cases} F_j^m(\xi,z)=A_j^m(\xi)e^{-\lambda_{j,1}z}+B_j^m(\xi)e^{-\lambda_{j,2}z}+C_j^m(\xi)e^{\lambda_{j,1}z}+D_j^m(\xi)e^{\lambda_{j,2}z}, \\ \chi_j^m(\xi,z)=G_j^m(\xi)e^{-\lambda_{j,3}z}+H_j^m(\xi)e^{\lambda_{j,3}z}, \qquad j\neq n+1 \end{cases}$$
$$\begin{cases} F_j^m(\xi,z)=A_j^m(\xi)e^{-\lambda_{j,1}z}+B_j^m(\xi)e^{-\lambda_{j,2}z}, \\ \chi_j^m(\xi,z)=G_j^m(\xi)e^{-\lambda_{j,3}z}, \qquad j=n+1 \end{cases} \tag{1}$$

$A_j^m(\xi)$ to $H_j^m(\xi)$, and $A_{n+1}^m(\xi)$, $B_{n+1}^m(\xi)$ and $G_{n+1}^m(\xi)$ in Eq. (1) are unknown functions which appeared due to integration. The continuity conditions and a relaxed treatment (see [1, 2]) of the boundary conditions can be stated as follows:

$$\sigma_{j+1,rz}(r,\theta,z_j^+)-\sigma_{j,rz}(r,\theta,z_j^-)=0, \quad \sigma_{j+1,z\theta}(r,\theta,z_j^+)-\sigma_{j,z\theta}(r,\theta,z_j^-)=0,$$
$$\sigma_{j+1,zz}(r,\theta,z_j^+)-\sigma_{j,zz}(r,\theta,z_j^-)=0, \quad u_{j+1}(r,\theta,z_j^+)-u_j(r,\theta,z_j^-)=0,$$
$$v_{j+1}(r,\theta,z_j^+)-v_j(r,\theta,z_j^-)=0, \quad w_{j+1}(r,\theta,z_j^+)-w_j(r,\theta,z_j^-)=0, \tag{2}$$

for $r\geq 0$ and $0\leq\theta<2\pi$, and

$$\begin{aligned} &\sigma_{1,rz}(r,\theta,z=0)=-P(r,\theta), \quad \sigma_{1,z\theta}(r,\theta,z=0)=-Q(r,\theta),\\ &\sigma_{1,zz}(r,\theta,z=0)=-R(r,\theta),\\ &\sigma_{1,rz}(r,\theta,z=0)=\sigma_{1,z\theta}(r,\theta,z=0)=\sigma_{1,zz}(r,\theta,z=0)=0,\\ &u_1(r,\theta,z=0)=\Delta_h\cos\theta, \quad v_1(r,\theta,z=0)=-\Delta_h\sin\theta,\\ &w_1(r,\theta,z=0)=\Delta_v, \end{aligned} \tag{3}$$

for $r<a$ and $0\leq\theta<2\pi$.

For simplicity, we define the vector $\mathbf{x}_j^m$ for the jth layer as

$$\mathbf{x}_j^m(\xi,z)=\left\{u_m^{m+1}+iv_m^{m+1} \quad u_m^{m-1}-iv_m^{m-1} \quad w_m^m \quad \sigma_{rzm}^{m+1}+i\sigma_{\theta zm}^{m+1} \quad \sigma_{rzm}^{m-1}-i\sigma_{\theta zm}^{m-1} \quad \sigma_{zzm}^m\right\}_j^T, \tag{4}$$

where the superscript T shows the transpose of the matrix. As seen in (4), the vector $\mathbf{x}^m$ is given in Fourier–Hankel transformed space. In this way, one may write this vector at $z=0$ as

$$\mathbf{x}_1^m(\xi,z=0)=\left\{u_m^{m+1}+iv_m^{m+1} \quad u_m^{m-1}-iv_m^{m-1} \quad w_m^m \quad -Y_m \quad -X_m \quad -Z_m\right\}_1^T, \tag{5}$$

where $X_m=P_m^{m-1}-iQ_m^{m-1}, Y_m=P_m^{m+1}+iQ_m^{m+1}$, and $Z_m=R_m^m$. The boundary conditions (3) imply that $X_1=Y_{-1}$, $X_{-1}=Y_1$, $X_m=Y_m=0$ for $m\neq\pm1$ and $Z_m=0$ for $m\neq0$. With the use of displacement- and stress-potential function relationships, the matrix $\mathbf{x}^m$ may be written as

$$\mathbf{x}_j^m=\mathbf{M}_j(\xi,z)\left[A^m \quad B^m \quad C^m \quad D^m \quad G^m \quad H^m\right]_j^T, \tag{6}$$

where $\mathbf{M}_j(\xi,z)$ is a 6×6 matrix. Thus, the vector $\mathbf{x}$ for the top and the bottom of the jth-layer can be written as $\mathbf{x}_{j,top}^m=\mathbf{M}_j(\xi,z_{j,top})\left[A^mB^mC^mD^mG^mH^m\right]_j^T$ and $\mathbf{x}_{j,bot}^m=\mathbf{M}_j(\xi,z_{j,bot})\left[A^mB^mC^mD^mG^mH^m\right]_j^T$. By substituting the vector $\left[A^m \quad B^m \quad C^m \quad D^m \quad G^m \quad H^m\right]j^T$ from $\mathbf{x}_{j,bot}^m$ into $\mathbf{x}_{j,top}^m$, the vector $\mathbf{x}_{j,top}^m$ is determined in terms of $\mathbf{x}_{j,bot}^m$ as $\mathbf{x}_{j,top}^m=\mathbf{T}_j\mathbf{x}_{j,bot}^m$where $\mathbf{T}_j=\mathbf{M}_j(\xi,z_{j,top})\mathbf{M}_j^{-1}(\xi,z_{j,bot})$ is the jth-layer transfer matrix connecting the variables at the top to the bottom. With the use of $\mathbf{T}$ and the continuity conditions $\mathbf{x}_{j,bot}^m=\mathbf{x}_{j+1,top}^m$, one may write

$$\mathbf{x}_{1,top}^m=\mathbf{T}_1\ \dots\ \mathbf{T}_n\ \mathbf{x}_{n+1,top}^m \tag{7}$$

Substituting $\mathbf{x}_{1,top}^m$ and $\mathbf{x}_{n+1,top}^m$, respectively from Eqs. (5) and (6) into Eq. (7) results in

$$\left\{u_m^{m+1}+iv_m^{m+1} \quad u_m^{m-1}-iv_m^{m-1} \quad w_m^m \quad -Y_m \quad -X_m \quad -Z_m\right\}_1^T=$$
$$(\mathbf{T}_1\dots\mathbf{T}_n\mathbf{M}_{n+1}(\xi,z_{n+1,top}))\left[A^m \quad B^m \quad 0 \quad 0 \quad G^m \quad 0\right]_{n+1}^T, \tag{8}$$

where because of the regularity conditions, the coefficients C^m, $D^m$and H^m for the lowest half-space are set as zero. Solving this equation results in

$$A_{n+1}^m=\frac{\mathbf{D}_{11}(X_m-Y_m)+\mathbf{D}_{13}Z_m}{\mathbf{D}}, \quad B_{n+1}^m=\frac{\mathbf{D}_{21}(X_m-Y_m)+\mathbf{D}_{23}Z_m}{\mathbf{D}},$$
$$G_{n+1}^m=\frac{\mathbf{D}_{31}(X_m+Y_m)}{\mathbf{D}}, \tag{9}$$

where $\mathbf{D}_{11}=-G_{45}G_{62}$, $\mathbf{D}_{12}=-\mathbf{D}_{11}$, $\mathbf{D}_{13}=2G_{45}G_{52}$, $\mathbf{D}_{21}=G_{45}G_{61}, \mathbf{D}_{22}=-\mathbf{D}_{21}$, $\mathbf{D}_{23}=2G_{41}G_{55}$, $\mathbf{D}_{31}=(G_{41}G_{62}-G_{42}G_{61})$, $\mathbf{D}_{32}=\mathbf{D}_{31}$, $\mathbf{D}=2G_{42}G_{45}G_{61}$, and $G_{ij}=[\mathbf{T}_1\dots\mathbf{T}_n(\mathbf{M}_{n+1}(\xi,z_{n+1,top}))]_{ij}$.

It is clear from this equation that $\mathbf{D}$ should be non-zero. Then, using $\mathbf{x}_{j,top}^m$and$\mathbf{x}_{j,bot}^m$ and the continuity conditions $\mathbf{x}_{j,bot}^m=\mathbf{x}_{j+1,top}^m$, one can write the following relationship between the coefficients