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Compressional and shear wave intrinsic attenuation and velocity in partially saturated soils

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ABSTRACT

A model for studying the propagation of compressional and shear waves in partially saturated soils is presented. The pores are filled with two immiscible fluids and the existence of four different wave modes including three compressional waves and one shear wave is demonstrated. The novel feature of the model is the consideration of tortuosity of fluid phases which are dependent on matric suction. The dispersion relations derived from the presented model are incorporated to study the influence of fluid saturation degree and frequency on the velocity and intrinsic attenuation of shear and compressional waves. Numerical simulations are performed on sand containing air–water mixture.

1. Introduction

A porous medium is composed of a matrix (skeleton) formed by solid particles and a porous space which can be filled by a single or several fluids (e.g. water, air, oil). Wave propagation phenomena in such media are investigated by several fields including seismology, geotechnics, geophysics and earthquake engineering. In all these fields of study, it is important to correctly predict and understand how the behavior of wave attributes like velocity and attenuation are affected by changes in characteristics of porous geomaterials.

Using the fundamentals of transport in porous media, Biot [1–3] developed the theory of wave propagation in porous media saturated only by a single compressible viscous fluid. Biot theory predicted the existence of three wave modes including two compressional and one shear wave. Biot's theory was widely used by the researchers in different fields [4–6] and various extensions to this theory were afterward proposed [7–10]. Brutsaert [11] extended the Lagrangian equations of Biot theory to account them for wave propagation in porous media containing two compressible fluids in their interstice. He showed that three compressional waves can theoretically exist in such media. However, relative acceleration between fluid phases and solid skeleton which causes inertial coupling was neglected in his study. Berryman [12] considered inertial coupling effects and generalized equations of motions to higher frequency range. However, he assumed capillary pressure changes are negligible during the passage of waves through a partially saturated medium. Santos et al. [13,14] used the principle of virtual complementary work and were first to obtain equations of motion for partially saturated porous media, including both capillary pressure and inertial effects. Further remarkable studies were carried out by Tuncay and Corapcioglu [15] who obtained macroscopic equations by volume averaging the microscale balance and constitutive equations and also Wei and Muraleetharan [16] who employed the mixture theory.

Despite all these achievements, a model that can consider the tortuosity of each fluid phase is not present. When a fluid flows through a porous medium, fluid particles follow a path with full of twists and turns. To describe this so called tortuous path, Carman [17] defined a parameter called tortuosity. When two immiscible fluids flow through a porous medium, each fluid follows a separate and distinct path. The distribution of fluids depends mainly on capillary pressure, viscous stresses on the interfaces and morphology of the pore spaces [18,19]. The direction of acceleration of each fluid may differ from the macroscopic acceleration direction due to the fact that tortuosity exists and tortuous path for each fluid phase is different. Considering this effect will lead to more realistic incorporation of inertial coupling mechanism between the pore fluids and the solid movements [20,21]. Therefore, the need for developing the wave motion equations capable of considering each phase tortuosity is necessary. The present study addresses

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this problem and also investigates the influence of changes in saturation and frequency on intrinsic attenuation and velocity of propagated waves in sand. The formulation presented in this study is developed for the case of air and water fluid mixtures but it is valid for all other nonwetting–wetting systems as well.

2. Governing equations

Following Fredlund and Morgenstern [22] and Fredlund and Rahardjo [23], stress–strain relations in unsaturated soils are obtained by Conte et al. [24] under the assumptions of isotropic linear elastic soil skeleton, infinitesimal strains, isothermal conditions and incompressible individual solid grains. These relations are written in terms of total stress tensor $\boldsymbol{\sigma}$, pore-water pressure p_w and air-pressure p_a as follows

$$\boldsymbol{\sigma} = G(\nabla \mathbf{u}_s + \nabla^T \mathbf{u}_s) + H\,\mathbf{I}\,\nabla\cdot\mathbf{u}_s + \phi_w[\chi\, L + (1-\chi\,)C]\mathbf{I}\,\nabla\cdot\mathbf{u}_w + \phi_a[\chi\, C + (1-\chi\,)N]\mathbf{I}\,\nabla\cdot\mathbf{u}_a \tag{1}$$

$$p_w = W\,\nabla\cdot\mathbf{u}_s - \phi_w L \nabla\cdot\mathbf{u}_w - \phi_a C\,\nabla\cdot\mathbf{u}_a \tag{2}$$

$$p_a = M\,\nabla\cdot\mathbf{u}_s - \phi_w C\,\nabla\cdot\mathbf{u}_w - \phi_a N\,\nabla\cdot\mathbf{u}_a \tag{3}$$

where T is transpose, $\mathbf{I}$ denotes identity tensor, G is shear modulus, χ is effective stress parameter and ϕ is the soil porosity. Displacement fields of the solid phase, water phase and air phase are denoted by $\mathbf{u}_s$, $\mathbf{u}_w$ and $\mathbf{u}_a$, respectively. Water volume fraction ϕ_w is related to the degree of water saturation S_w by the relation $\phi_w = \phi S_w$ and air volume fraction ϕ_a is derived by $\phi_a = \phi\,(1-S_w)$. Other parameters appearing in Eqs. (1)–(3) are given by

$$H = 2a_s G - \chi W - (1-\chi)M \tag{4}$$

$$W = -\frac{K_w A}{D} + \phi_w L + \phi_a C \tag{5}$$

$$M = -\frac{K_a B}{D} + \phi_w C + \phi_a N \tag{6}$$

$$L = \frac{K_w[\phi_a m_1^s + K_a(m_1^s m_2^w - m_2^s m_1^w)]}{D} \tag{7}$$

$$N = \frac{K_a[\phi_w m_1^s + K_w(m_1^s m_2^w - m_2^s m_1^w)]}{D} \tag{8}$$

$$C = \frac{K_w K_a(m_1^s m_2^w - m_2^s m_1^w)}{D} \tag{9}$$

$$A = \phi_a m_1^w + K_a(m_1^s m_2^w - m_2^s m_1^w) \tag{10}$$

$$B = \phi_w m_1^a + K_w(m_1^s m_2^w - m_2^s m_1^w) \tag{11}$$

$$D = (m_1^s m_2^w - m_2^s m_1^w)(\phi_a K_w + \phi_w K_a) + \phi_a \phi_w m_1^s \tag{12}$$

where a_s is related to Poisson's ratio of soil skeleton by $a_s = \upsilon/1-2\upsilon$, K_w is water bulk modulus, K_a is air bulk modulus, m_1^s is coefficient of volume change with respect to net normal stress, m_2^s is coefficient of volume change with respect to matric suction, m_1^w is coefficient of water volume change with respect to net normal stress, m_2^w is coefficient of water volume change with respect to matric suction. Coefficients of air volume changes m_1^a and m_2^a are related to m_1^s,m_2^s,m_1^w and m_2^w by the relations $m_1^s = m_1^w + m_1^a$ and $m_2^s = m_2^w + m_2^a$. The effective stress parameter χ can be derived by the relation $\chi = m_2^s/m_1^s$. It is useful to note that Cosentini [25] showed $m_1^w = m_2^s$.

To derive wave equations of motion, the momentum conservation equations should be introduced. Momentum equation expressing the equilibrium of all forces acting on an elementary volume of medium is expressed by [24]

$$\nabla\cdot\boldsymbol{\sigma} - \rho_s(1-\phi)\partial_t^2\mathbf{u}_s - \rho_w\phi_w\partial_t^2\mathbf{u}_w - \rho_a\phi_a\partial_t^2\mathbf{u}_a = 0 \tag{13}$$

Since the primary objective is to consider the inertial coupling effects caused by the tortuosity of fluid phases, unsteady terms should be added into Darcy's law. Therefore, we use fluid momentum equations in the form presented by Smeulders [26] to describe the pore-water and pore-air movement through the soil

$$\phi_w\rho_w\partial_t^2\mathbf{u}_w = -\phi_w\nabla p_w + (\tau_w-1)\phi_w\rho_w\partial_t^2(\mathbf{u}_s-\mathbf{u}_w) + b_w\partial_t(\mathbf{u}_s-\mathbf{u}_w) \tag{14}$$

$$\phi_a\rho_a\partial_t^2\mathbf{u}_a = -\phi_a\nabla p_a + (\tau_a-1)\phi_a\rho_a\partial_t^2(\mathbf{u}_s-\mathbf{u}_a) + b_a\partial_t(\mathbf{u}_s-\mathbf{u}_a) \tag{15}$$

where τ_w and τ_a are effective tortuosity of water and air phases. Parameters b_w and $b_a$are derived by

$$b_w = \frac{{\phi_w}^2\eta_w}{k_w} \tag{16}$$

$$b_a = \frac{{\phi_a}^2\eta_a}{k_a} \tag{17}$$

where η_w is the water viscosity, η_a is the air viscosity and k_w and k_a are the effective permeability of water and air phases, respectively. Effective permeability is defined as the permeability of a porous medium to a particular fluid phase when more than one fluid phase is present in the pore spaces.

Finally, partial differential equations of wave motion in unsaturated soils are derived from constitutive Eqs. (1)–(3) and momentum Eqs. (13)–(15) after performing some algebraic manipulations

$$\zeta\,\nabla\nabla\cdot\mathbf{u}_s - G(\nabla\nabla\cdot\mathbf{u}_s - \nabla^2\mathbf{u}_s) + \varsigma\,\nabla\nabla\cdot\mathbf{u}_w + \xi\,\nabla\nabla\cdot\mathbf{u}_a - (\tau_{f1}-1)\phi_w\rho_w\partial_t^2(\mathbf{u}_s-\mathbf{u}_w) - (\tau_{f2}-1)\phi_a\rho_a\partial_t^2(\mathbf{u}_s-\mathbf{u}_a) - b_w\partial_t(\mathbf{u}_s-\mathbf{u}_w) - b_a\partial_t(\mathbf{u}_s-\mathbf{u}_{f2}) - \rho_s(1-\phi)\partial_t^2\mathbf{u}_s = 0 \tag{18}$$

$$-\phi_w\,W\,\nabla\nabla\cdot\mathbf{u}_s + (\phi_w)^2 L\,\nabla\nabla\cdot\mathbf{u}_w + \phi_w\phi_a\,C\,\nabla\nabla\cdot\mathbf{u}_a + (\tau_w-1)\phi_w\rho_w\partial_t^2(\mathbf{u}_s-\mathbf{u}_w) + b_w\partial_t(\mathbf{u}_s-\mathbf{u}_w) - \phi_w\rho_w\partial_t^2\mathbf{u}_s = 0 \tag{19}$$

$$-\phi_a\,M\,\nabla\nabla\cdot\mathbf{u}_s + (\phi_a)^2 N\,\nabla\nabla\cdot\mathbf{u}_a + \phi_w\phi_a\,C\,\nabla\nabla\cdot\mathbf{u}_w + (\tau_a-1)\phi_a\rho_a\partial_t^2(\mathbf{u}_s-\mathbf{u}_a) + b_a\partial_t(\mathbf{u}_s-\mathbf{u}_a) - \phi_a\rho_a\partial_t^2\mathbf{u}_s = 0 \tag{20}$$

where

$$\zeta = H + 2G + W\phi_w + M\phi_a \tag{21}$$

$$\varsigma = \phi_w[\chi L + (1-\chi)C] - (\phi_w)^2 L - \phi_w\phi_a C \tag{22}$$

$$\xi = \phi_a[\chi C + (1-\chi)N] - (\phi_a)^2 N - \phi_w\phi_a C \tag{23}$$

Note that by putting $\tau_w = \tau_a = 1$, the above equations coincide with the wave equations of motion presented in Conte et al. [24].

3. Dispersion relations

3.1. Decomposition

The solid phase displacement vector $\mathbf{u}_s(x,t)$, the pore-water phase displacement vector $\mathbf{u}_w(x,t)$ and the pore-air phase displacement vector $\mathbf{u}_a(x,t)$ can be decomposed in terms of the compressional (longitudinal) wave potentials Φ_α ($\alpha = s,\ w,\ a$) and shear (transverse) wave potentials Ψ_α ($\alpha = s,\ w,\ a$) by using Helmholtz decomposition as given below [27]:

$$\mathbf{u}_s = \nabla\Phi_s + \nabla\times\Psi_s \tag{24}$$

$$\mathbf{u}_w = \nabla\Phi_w + \nabla\times\Psi_w \tag{25}$$