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Three-dimensional simulation of track on poroelastic half-space vibrations due to a moving point load

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ABSTRACT

An analytical approach is used to investigate dynamic responses of a track system and the poroelastic half-space soil medium subjected to a moving point load under three-dimensional condition. The whole system is divided into two separately formulated substructures, the track sub-system and the ground. The ballast supporting rails and sleepers is placed on the surface of the ground. The rail is modeled by introducing the Green function for an infinitely long Euler beam subjected to the action of the moving point load and the reaction of sleepers represented by a continuous mass. Using the double Fourier transform, the governing equations of motion are then solved analytically in the frequency–wave-number domain. The time domain responses are evaluated by the inverse Fourier transform computation for a certain load velocities. Computed results show that dynamic responses of the soil medium are considerably affected by the fluid phase as well as the load velocity.

1. Introduction

Vibration assessments alongside railway tracks are becoming of importance as a result of increasing speeds of modern trains. Particularly, these problems are significantly important if the track is laid on the ground of soft clay in which the shear wave velocity may be as low as 40 m/s.

The three-dimensional problem of the steady-state motion of a point load moving on the free surface has been considered by Papadopoulos [1] and Eason [2] with different approaches. Gakenheimer and Miklowitz [3] presented the transient responses in the interior of the elastic half-space for a suddenly applied point load which then moves with a constant velocity along the free surface. A closed-form solution for the normal displacement of the surface of the half-space subjected to a normal point force moving at a constant velocity on the surface of an elastic half-space was given by Barber [4]. Barros and Luco [5] extended the studies to the dynamic responses of a multi-layered visco-elastic half-space generated by a buried point load moving parallel to the surface of the half-space or a surface moving point load. Cai et al. [6] and Liu et al. [7] studied the dynamic responses of a poroelastic half-space under moving rectangular load using different method. Vibration of a track system on the ground was first considered by Kennedy and Herrmann [8,9], who gave an analytical solution of infinite beam on the visco-elastic ground under moving point load. Hung and Yang [10] presented a work dealing with the dynamic responses of an elastic soil medium subjected to a moving point load distributed by an elastic beam. Kargarnovin et al. [11] and Kargarnovin and Younesian [12] investigated the response of a Timoshenko beam on the Pasternak visco-elastic half-space and nonlinear visco-elastic half space, respectively. Sun [13] gave a closed-form solution of beam on the visco-elastic subgrade subjected to a moving point load. Sheng et al. [14] extended the investigations to the more realistic layered ground system. Takemiya and Bian [15] and Picoux and LeHouedec [16] studied the dynamic responses of a track system on the ground subjected to a moving train load. Generally, there is under-ground water in the considered soil medium, but the study of the dynamic responses of a track system laying on a poroelastic soil medium are rather limited. Jin [17] studied the responses of an Euler beam on a poroelastic half-space subjected to a moving point load. The existence of the water affects the wave propagation in the soil medium in some degree during the passage of the train, and the track system distributes the load in a different way to an Euler beam. The fully saturated poroelastic soil with a track system model is a good choice to be used for the analysis of dynamic responses of the track-ground system.

In this paper, an analytical approach is used to investigate the dynamic responses of a track system and poroelastic half-space soil medium subjected to moving point load. The whole system is divided into two separately formulated substructures, the track and the ground. On top of the ground, ballast is placed, supporting the rails and the sleepers. The rail is modeled by introducing the

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Green function for an infinitely long Euler beam subjected to the action of moving axle load and the reactions of the sleeper. Sleepers are represented by a continuous mass. Using the double Fourier transform, the governing equations of motion are then solved analytically in the frequency–wave-number domain. The time domain responses are evaluated by the inverse Fourier transform computation for a certain load velocities. Computed results show that dynamic responses of the soil medium are considerably affected by the fluid phase as well as the load velocity. Computed results of the proposed model are compared with those of existing methods.

2. Governing equations and general solutions

2.1. Analytical solution of poroelastic half space

The system considered herein, shown in Fig. 1, consists of a uniform porous elastic solid material, fully saturated by the viscous fluid, and extended to infinity in x, y, z directions. The fluid is free to flow throughout the entire upper surface. The rail is described by introducing the Green function for an infinite long Euler beam subjected to the action of the moving point load and the reactions of the sleeper. Sleepers are represented by a continuous mass and the effect of the ballast is considered by introducing the Cosserat model for granular medium.

Based on the assumption of neglecting the apparent mass density, the linearized dynamic equations of motion of a fully saturated poroelastic are given by Biot [18] as:

$$\mu u_{i,jj}+(\lambda+\alpha^2 M+\mu)u_{j,ji}+\alpha M w_{j,ji}=\rho\ddot{u}_i+\rho_f\ddot{w}_i \tag{1}$$

$$\alpha M u_{j,ji}+M w_{j,ji}=\rho_f\ddot{u}_i+m\ddot{w}_i+b\dot{w}_i \tag{2}$$

where, u_i, w_i (i=x, y, z) are the solid displacement components and fluid displacement related to solid displacement along the x, y, z directions; dots on u_i and w_i indicate the differential with respect to time t; λ and μ are Lame constants; α and M are Biot's parameters accounting for compressibility of the two-phased material; ρ and ρ_f are the actual mass densities of the solid and the fluid, respectively; m is a density-like parameter that depends on ρ_f and the geometry of the pores; b is a parameter accounting for the internal friction due to the relative motion between the solid and the pore fluid. The parameter b equals to the ratio between the fluid viscosity and the intrinsic permeability of the medium (b=0 for the neglection of internal friction). The constitutive relations can be expressed as:

$$\sigma_{ij}=\lambda\delta_{ij}\theta+\mu(u_{i,j}+u_{j,i})-\alpha\delta_{ij}p \tag{3}$$

$$p=-\alpha M\theta+M\varsigma \tag{4}$$

where

$$\varsigma=-w_{i,i} \tag{5}$$

and $\theta=u_{i,i}$ is solid strain; σ_{ij} is the total stress component of bulk material; p is the pore water pressure.

In this paper, the dimensionless coordinates are defined as: x^*=x/a, y^*=y/a, z^*=z/a and the dimensionless time is defined as $\tau=(t/a)\sqrt{\mu/\rho}$, in which a is the half width of the railway track.

The Fourier transform with respect to dimensionless time τ is defined as

$$\tilde{f}(x^*,y^*,z^*,\Omega)=\int_{-\infty}^{\infty}f(x^*,y^*,z^*,\tau)e^{-i\Omega\tau}d\tau \tag{6}$$

and the inverse relationship is given by

$$f(x^*,y^*,z^*,\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{f}(x^*,y^*,z^*,\Omega)e^{i\Omega\tau}d\Omega \tag{7}$$

By making use of Eq. (6) and after some manipulations, Eqs. (1)–(5) lead to the following form:

$$\nabla^2\tilde{u}_x+(\lambda^*+1)\partial\frac{\tilde{\theta}}{\partial x}+\Omega^2(1-\rho^*\vartheta)\tilde{u}_x-(\alpha-\vartheta)\partial\frac{\tilde{p}}{\partial x}=0 \tag{8}$$

$$\nabla^2\tilde{u}_y+(\lambda^*+1)\partial\frac{\tilde{\theta}}{\partial y}+\Omega^2(1-\rho^*\vartheta)\tilde{u}_y-(\alpha-\vartheta)\partial\frac{\tilde{p}}{\partial y}=0 \tag{9}$$

$$\nabla^2\tilde{u}_z+(\lambda^*+1)\partial\frac{\tilde{\theta}}{\partial z}+\Omega^2(1-\rho^*\vartheta)\tilde{u}_z-(\alpha-\vartheta)\partial\frac{\tilde{p}}{\partial z}=0 \tag{10}$$

$$\nabla^2\tilde{p}+\frac{\rho^*\Omega^2}{M^*\vartheta}\tilde{p}+\frac{\rho^*\Omega^2(\alpha-\vartheta)}{\vartheta}\tilde{\theta}=0 \tag{11}$$

The dimensionless parameters in the former equations are defined as: $\lambda^*=\lambda/\mu$, $M^*=M/\mu$, $\rho^*=\rho_f/\rho$, $m^*=m/\rho$, $b^*=ab/\sqrt{\rho\mu}$, $\vartheta=\rho^*\Omega^2/(m^*\Omega^2-ib^*\Omega)$ and ∇^2 is the Laplacian operator defined by:

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

From Eqs. (8)–(11) the following equation can be obtained:

$$\nabla^4\tilde{p}+a_1\nabla^2\tilde{p}+a_2\tilde{p}=0 \tag{12}$$

in which

$$a_1=\frac{(m^*\Omega^2-ib^*\Omega)(\lambda^*+\alpha^2M^*+2)+M^*\Omega^2-2\alpha M^*\rho^*\Omega^2}{(\lambda^*+2)M^*} \tag{13}$$

$$a_2=\frac{(m^*\Omega^2-ib^*\Omega)\Omega^2-(\rho^*)^2\Omega^4}{(\lambda^*+2)M^*} \tag{14}$$

The Fourier transforms with respect to the dimensionless x- and y-coordinates is defined as

$$\bar{\bar{f}}(\xi,\eta,z^*,\Omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x^*,y^*,z^*,\Omega)e^{-i(\xi x^*+\eta y^*)}dxdy \tag{15}$$

and the inverse relationship is given by

$$f(x^*,y^*,z^*,\Omega)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{\bar{f}}(\xi,\eta,z^*,\Omega)e^{i(\xi x+\eta y)}d\xi d\eta \tag{16}$$

The application of Fourier integral transforms to Eqs. (3), (4), (8)–(11) lead to:

$$\bar{\bar{p}}(\xi,\eta,z^*,\Omega)=A(\xi,\eta,\Omega)e^{-\gamma_1 z^*}+B(\xi,\eta,\Omega)e^{-\gamma_2 z^*} \tag{17}$$

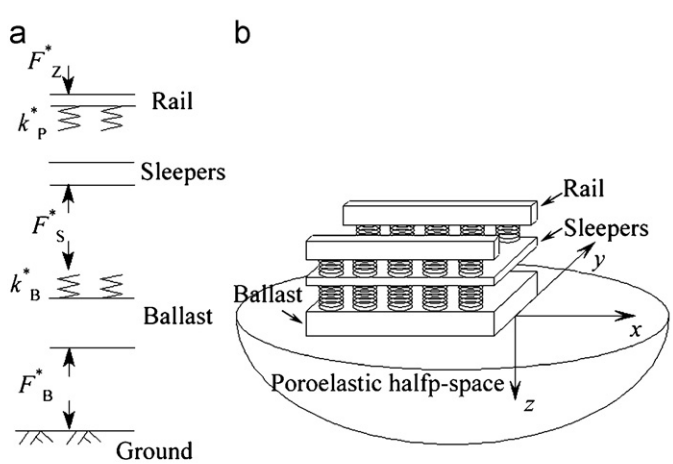

Fig. 1. (a) Rail-sleeper–ballast-ground interaction model. (b) Track system on a poroelastic half-space.