Soil Dynamics and Earthquake Engineering 30 (2010) 133–145

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Structural damage detection using empirical-mode decomposition and vector autoregressive moving average model

Dong Yinfeng[a,*], Li Yingmin[a], Lai Ming[b]

[a] College of Civil Engineering, Chongqing University, Chongqing 400045, China
[b] Department of Science and Technology, Ministry of Construction, Beijing 100835, China

ARTICLE INFO

Article history:
Received 16 February 2009
Received in revised form
4 October 2009
Accepted 6 October 2009

Keywords:
Damage detection
Signal processing
Vibration
Empirical-mode decomposition
Vector autoregressive moving average model

ABSTRACT

A method based on empirical-mode decomposition (EMD) and vector autoregressive moving average (VARMA) model is proposed for structural damage detection. The basic idea of the method is that the structural damages can be identified as the abrupt changes in energy distribution of structural responses at high frequencies. Using the time-varying VARMA model to represent the intrinsic mode functions (IMFs) obtained from the EMD of vibration signal, we define a damage index according to the VARMA coefficients. In the two examples given, the Imperial County Services Building and the Van Nuys hotel are used as the benchmark structures to verify the effectiveness and sensitivity of the damage index in real environments with the presence of actual noise. The analysis results show that the damage index can indicate the occurrence and relative severity of structural damages at multiple locations in an efficient manner. The damage index can also be potentially used for structural health monitoring, since it is based on the time-varying VARMA coefficients. Finally, some recommendations for future research are provided.

1. Introduction

Structural damage detection is an important and challenging issue in earthquake engineering. As a systemic research field, it involves the interaction and integration of many disciplines such as signal processing, stochastic process theory, structural dynamic analysis, system identification, sensor technology, and numerical simulation. The main purpose of structural damage detection is to identify the occurrence (presence), location and type of damage to quantify the damage severity and to predict the remaining service life of the structure. Vibration data obtained from instrumented structures are often used for this purpose.

Since last three decades, many damage detection methods have been developed. Excellent review work on this topic is found in [1–4]. The most commonly used damage detection methods can be roughly divided into two categories. One is the method that directly estimates the physical parameters such as stiffness or damping ratios, and investigates their changes [1,5,6]. The finite element (FE) model updating is one of the most frequently used methods in this category. It uses the test (measured) data of structure as a reference quantity, and modifies the mass, stiffness and damping parameters of the corresponding FE model to obtain best agreement between numerical results and test data. Then, the geometric location and severity of structure damage is determined according to the modification coefficients. To obtain the modification coefficients, optimum algorithm is often used with test data as objective functions. The other category mainly consists of various vibration-based methods that monitor the changes in modal parameters (modal frequency, mode shape and modal damping). The basic premise of these methods is that modal parameters are functions of the physical properties of structure (mass, damping, stiffness and boundary conditions). Therefore, the changes in physical parameters of structure will cause changes in modal properties. Modal parameters are, in many cases, easier to evaluate in the field; this category of methods is more popular than the first one. And the modal parameters, such as natural frequency, mode shape curvatures, modal flexibility and its derivatives, modal stiffness, modal strain energy (MSE), frequency response function (FRF), FRF curvatures, modal assurance criterion (MAC), coordinate modal assurance criterion (COMAC) and multiple damage location assurance criterions (MDLAC), are frequently used in practical applications [1–4]. However, the effectiveness of this category is usually limited by the configuration of sensors, parameter estimation error, measurement noise, and the sensitivity of modal parameters to structural damages. Continuous efforts to reduce such limitations have given rise to some improved methods [7–13]. In addition, some more recent research has shown that extracting physically meaningful information from vibration signals to detect structural damages may be the new trend [14–25], which

* Corresponding author. Tel.: +86 23 6512 1991; fax: +86 23 6512 3511.
E-mail addresses: yhvhson@cta.cq.cn, dongyinfeng@cqu.edu.cn (Y. Dong).

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doi:10.1016/j.soildyn.2009.10.002

undoubtedly requires some more efficient methods to process the vibration signals that are often nonlinear and nonstationary in nature.

Among various signal processing methods the Hilbert–Huang transform (HHT) developed by Huang et al. [26,27], is thought to be particularly suitable for processing nonlinear and nonstationary signals. HHT consists of empirical-mode decomposition (EMD) and Hilbert spectral analysis (HSA). The purpose of EMD is to decompose a signal into a finite set of intrinsic mode functions (IMFs), which admit well-behaved Hilbert transforms. HSA is designated to calculate the instantaneous frequencies and amplitude of IMFs through the Hilbert transform and to obtain the time–frequency distribution of amplitude called Hilbert spectrum. Though HHT has been successfully used in many fields since its introduction [26,28–31], there still exist two limitations associated with the restriction of the Nuttall theorem and larger covariance (poorer frequency resolution and readability) of the Hilbert spectrum at higher frequencies [32–34]. To address the above problems, a time-varying vector autoregressive moving average (VARMA) model-based method was proposed in [34], and the preliminary results showed the method was quite effective.

As a further application of the improved HHT method, a damage index derived from the time-varying VARMA coefficients is proposed in this study to detect the structural damages. The technical details of HHT have been reported in many literatures. Therefore, only the improved HHT method is briefly reviewed in the following section. Then, the damage index is described in detail. In the two examples given, the Van Nuys hotel [20,21,41] and the Imperial County Services Building [22–24] are used as the benchmark structures to explore the effectiveness of the proposed damage index. The recorded seismic responses are analyzed using both the improved and the original HHT methods, and the proposed damage index is applied to detect the structural damages. Based on the damage detection results, the effectiveness and limitations of the proposed method are discussed, and some recommendations for future research are provided.

2. The improved HHT method

To circumvent the limitations of HHT, we have proposed a time-varying VARMA model-based method to calculate the instantaneous frequencies of IMFs and yield the Hilbert spectrum [34]. The improved method is summarized as follows:

1. Perform the EMD to get all the n IMFs for a given signal $x(k)$, which is the same as the original HHT method.
2. Represent the n IMFs as a time-varying VARMA (p, q) model

$$\boldsymbol{y}(k)=\sum_{i=1}^{p}\Phi_i(k)\boldsymbol{y}(k-i)+\sum_{i=1}^{q}\Theta_i(k)\mathbf{u}(k-i)+\mathbf{u}(k), \tag{1}$$

where the vector $\boldsymbol{y}(k)$ consists of the n IMFs, i.e. $\boldsymbol{y}(k)=[c_1(k), c_2(k), \ldots, c_n(k)]^T{}_{n\times 1}$ and $c_i(k)$ is the ith IMF. The autoregressive coefficients $\Phi_i(k)$ and the moving average coefficients $\Theta_i(k)$ are $n\times n$ matrices. The vector $\mathbf{u}(k)$ is an n-dimensional zero-mean Gaussian white noise process with covariance matrix $R(k)\mathbf{I}$, where $\mathbf{I}$ is the unit matrix and $R(k)>0$. The variable k indicates the time instant $t=k\Delta t$ with Δt as the sampling interval. It has been shown by Pandit [35] and Anderson [36] that a time-varying n-dimensional VARMA ($2m$, $2m-1$) model is equivalent to a time-varying system with nm degrees of freedom (DOF). As all the n IMFs are monocomponent signals, they can be viewed as the n outputs of a time-varying n-DOF system, and the n instantaneous eigenfrequencies (natural frequencies) of the system can be designated as the instantaneous frequencies of the IMFs. Consequently, a time-varying n-dimensional VARMA (2, 1) model can be used to represent the n IMFs.
3. Recast the time-varying VARMA (p, q) model into state space form

$$\begin{cases}\xi(k)=\xi(k-1)+\mathbf{v}(k-1),\\ \mathbf{y}(k)=[\mathbf{H}(k)\xi(k)]^T+\mathbf{u}(k),\end{cases} \tag{2}$$

where $\xi(k)=[\Phi_1(k),\ldots,\Phi_p(k),\ \Theta_1(k),\ldots,\Theta_q(k)]^T_{(np+nq)\times n}$ is called state vector. $\boldsymbol{v}(k)=[\mathbf{v}_1(k),\ \mathbf{v}_2(k),\ldots,\mathbf{v}_{p+q}(k)]^T_{(np+nq)\times n}$ is Gaussian process noise and its mean and covariance are $\mathbf{0}$ and $\mathbf{Q}(k)$ respectively. $\mathbf{Q}(k)$ is an symmetric positive definite matrix. $\mathbf{H}(k)=[\mathbf{y}(k-1)^T,\ldots,\mathbf{y}(k-p)^T,\ \mathbf{u}(k-1)^T,\ \ldots,\mathbf{u}(k-q)^T]_{1\times(np+nq)}$ is called measurement vector.
4. Use the Kalman filter to estimate $\xi(k)$ based on the above state space model. For more details about the improved method and the algorithm of Kalman filter, the readers can refer to [18,34,37,38].
5. Define the system matrix $\mathbf{A}(k)$ as

$$\mathbf{A}(k)=\begin{bmatrix}\mathbf{\Phi}_1(k) & \mathbf{\Phi}_2(k) & \cdots & \mathbf{\Phi}_p(k)\\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & & \mathbf{0}\\ \vdots & \vdots & & \vdots\\ \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0}\end{bmatrix}_{np\times np} \tag{3}$$

and decompose $\mathbf{A}(k)$ by taking eigenvalue decomposition

$$\mathbf{A}(k)=\Psi(k)\lambda(k)\Psi(k)^{-1}, \tag{4}$$

where $\lambda(k)=diag[\lambda_i(k)]$ for $i=1, 2, \ldots, np$. Then, the instantaneous eigenfrequencies of the system are given as

$$f_i(k)=||\ln[\lambda_i(k)]||/2\pi\Delta t. \tag{5}$$

Because the instantaneous eigenvalues $\lambda_i(k)$ appear in complex conjugate pairs, one pair for each degree of freedom, we can get n different instantaneous eigenfrequencies when $p=2$, i.e. the instantaneous frequencies of the n IMFs.
6. For each IMF, define the envelope by a cubic spline through all the maxima. Then use all the envelopes and instantaneous frequencies to obtain the Hilbert spectrum.

Because all the IMFs are monocomponent signals, the time-varying VARMA (2, 1) model is the optimal model. We do not need to select the model order according to the commonly used Akaike information criterion (AIC) or other criteria, which makes the method easy to implement. Moreover, using a time-varying VARMA model to represent the IMFs as the outputs of a time-varying system with white noise inputs, the method is more physically meaningful and can reduce the effect of the noise yielded during the EMD procedure.

3. The damage index

As has been described above, when an n-dimensional VARMA (2, 1) model is used in Eq. (1) to represent $\boldsymbol{y}(k)$ as the n outputs of an n-DOF system to input $\boldsymbol{u}(k)$, the n pair of instantaneous eigenvalues $\boldsymbol{\lambda}(k)$ yield n different instantaneous mode frequencies as given in Eq. (5). Similar to the instantaneous eigenvalues $\boldsymbol{\lambda}(k)$, the instantaneous eigenvectors $\mathbf{\Psi}(k)=[\mathbf{\Psi}_1(k),\ \mathbf{\Psi}_2(k),\ldots,\mathbf{\Psi}_{2n}(k)]_{2n\times 2n}$ appear in complex conjugate pairs, one pair for each mode, thus we can get n different instantaneous mode shapes of the system. For the ith mode, the instantaneous mode shape is defined as the last n components of the instantaneous eigenvector $\mathbf{\Psi}_{2i-1}(k)$ [18,34–36].

In this study, the time-varying VARMA (2, 1) model is used to represent all the n IMFs of a given signal $x(k)$ as the outputs of a time-varying system to white noise inputs, and $x(k)$ is taken as the