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Transmitting boundary for water-saturated transversely isotropic strata based on the *u–U* formulation

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ABSTRACT

A new transmitting boundary in a cylindrical coordinate system has been developed for modeling the elastic waves radiating out to an infinite boundary in water-saturated transversely isotropic soil strata over a rigid bedrock. The saturated soil strata are assumed to consist of a porous material and modeled as a transversely isotropic two-phase medium, based on the u–U formulation. The newly developed transmitting boundary is combined with the finite-elements model of the near-field region, using the same u–U formulation, and applied to the study of the dynamics of a rigid circular foundation in porous isotropic or transversely isotropic layered strata, either fully or partly saturated with water. The verification and application examples give valuable insights into new and interesting aspects of the dynamic behavior of rigid circular foundations in fully or partly saturated two-phase ground in terms of permeability, transverse anisotropy, and ground-water table level.

1. Introduction

The effects of soil–structure interaction can significantly influence the dynamic response of massive structures installed in relatively flexible ground. Dynamic soil–structure interaction has been studied extensively in connection with seismic design of structures. In most past studies, the ground was assumed to be either a layered half-space or layered strata over a rigid bedrock. The far-field region of the ground, where elastic waves radiate out to an infinite boundary, was modeled as a homogeneous medium either in the radial or the horizontal direction. The behavior of the rigid foundation in the layered strata over a rigid bedrock has been widely studied, partly due to the relative ease of mathematical modeling and analysis.

Waas developed a semi-analytic finite-element method for the analysis of wave propagation in layered strata over a rigid bedrock, and studied the behavior of rigid strip and circular foundations [1]. In his study, the near-field region was modeled using the conventional finite elements, with the semi-analytic finite-element method used for the far-field region, which is later referred to as a 'hyperelement'. Kausel extended Waas' work to develop a hyperelement for the non-axisymmetric horizontal and rocking motions of circular foundations [2]. Tassoulas developed both homogeneous and inhomogeneous circular and ring hyperelements by combining homogeneous and particular solutions that satisfied inhomogeneous boundary conditions in the near-field region [3]. General 3-D consistent boundaries were developed in a cylindrical coordinate system by Werkle [4] and by Lin and Tassoulas [5]. Recently, Kim et al. [6] extended this method further to problems in a general 3-D Cartesian coordinate system.

Since soil can be saturated with water below the underground-water table located at a certain depth, the ground must be modeled as a water-saturated two-phase porous medium. Hence the semi-analytic method was extended further to model the water-saturated strata. Using Biot's theory, Nogami and Kazama [7,8] formulated a thin layer method for dynamic response analysis of submerged isotropic soil. In their studies, not only the movement of solid particles but also the lateral flow of the fluid relative to solid skeleton was assumed to be fixed at the interface between soil and bedrock. Independently, Bougacha et al. [9,10] presented transmitting boundaries for the dynamic stiffness of a rigid strip and circular foundation on fluid-filled poroelastic isotropic strata. The formulation used in Bougacha et al. was slightly different from that of Nogami and Kazama.

The semi-analytic method is generalized further in this study. Since the ground usually has a stratified structure formed through a long geologic process and is affected by gravitation, it is quite reasonable to assume that the ground would consist of a transversely isotropic material rather than an isotropic one. In this study, soil strata are still assumed to be a water-saturated porous medium but modeled as a transversely isotropic one. A corresponding transmitting boundary in a cylindrical coordinate

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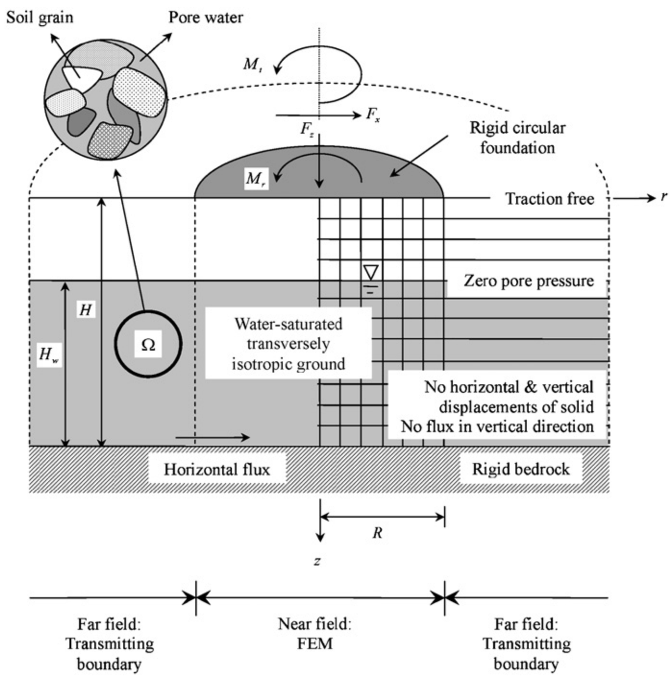

Fig. 1. Rigid circular foundation in water-saturated transversely isotropic layered strata over a rigid bedrock.

system is newly developed for modeling elastic waves radiating out to an infinite boundary in the water-saturated transversely isotropic soil strata over a rigid bedrock (Fig. 1). The lateral flow of fluid relative to the solid skeleton is permitted at the bed rock surface in this study. This boundary condition more closely represents the reality than the fixed condition adopted by Nogami and Kazama [7,8].

The newly developed transmitting boundary is combined with the finite-element model of the near-field region and applied to analyze dynamics of rigid circular surface foundations. The results are compared with existing analysis results by different models for the purpose of verification. Then the effects of permeability, transverse isotropy, and depth of the ground-water table on the dynamic stiffness of the foundation are studied.

2. Equations of motion for a water-saturated transversely isotropic medium

Governing equations for a water-saturated transversely isotropic porous medium can be derived using the generalized Biot's theory [11–14], and identical equations can be derived from the mixture theory [15]. If nonlinear convective terms, effects of the body force, and density and temperature changes are not taken into account, the equations in orthogonal curvilinear coordinate systems can be written as follows for a domain Ω (Fig. 1):

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\mathbf{u}} + \rho_w \ddot{\mathbf{w}} \tag{1a}$$

$$\nabla p + \mathbf{f}\dot{\mathbf{w}} + \rho_w \ddot{\mathbf{u}} + \frac{\rho_w}{n}\ddot{\mathbf{w}} = 0 \tag{1b}$$

$$\frac{1}{Q}\dot{p} + (\boldsymbol{\alpha}\nabla)\cdot\dot{\mathbf{u}} + \nabla\cdot\dot{\mathbf{w}} = 0 \tag{1c}$$

where $\boldsymbol{\sigma}$ denotes the total stress in the mixture of solid and fluid; $\mathbf{u}$ the displacement of the solid skeleton; $\mathbf{w} = n(\mathbf{U}-\mathbf{u})$ the 'pseudo-displacement' of the fluid with respect to the solid skeleton so defined that $\dot{\mathbf{w}}$ is the average relative velocity of seepage in which $\mathbf{U}$ is the displacement of the fluid; p the pore pressure in the fluid; n the porosity; ρ_w the density of the fluid; $\rho = (1-n)\rho_s + n\rho_w$ the averaged density of the mixture in which ρ_s is the density of the solid; and $\mathbf{f} = \boldsymbol{\kappa}^{-1}$, in which $\boldsymbol{\kappa}$ is the permeability tensor. Tensor $\boldsymbol{\kappa}$ can be expressed in a transversely isotropic medium, as follows:

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_h & & \\ & \kappa_h & \\ & & \kappa_v \end{bmatrix} \tag{2}$$

where κ_h and κ_v denote the permeability in isotropic and perpendicular planes, respectively. In Eq. (1c), $\boldsymbol{\alpha}$ is the generalized Biot's constant. It can be expressed for a transversely isotropic medium, as follows:

$$\boldsymbol{\alpha} = \mathbf{I} - \frac{1}{3K_s}\mathbf{D}[\mathbf{I}] = \begin{bmatrix} \alpha_h & & \\ & \alpha_h & \\ & & \alpha_v \end{bmatrix} \tag{3}$$

where $\mathbf{D}$ is the elastic constitutive tensor for a transversely isotropic medium, as shown in Appendix A; $\mathbf{I}$ the second-order identity tensor and K_s the bulk modulus of the solid grain. The term $\mathbf{D}[\mathbf{I}]$ means the product of the fourth-order tensor $\mathbf{D}$ and the second-order tensor $\mathbf{I}$. The bulk modulus Q in Eq. (1c) can be expressed in terms of the bulk modulus of the solid skeleton K_T, the bulk modulus of the solid grain K_s, and the bulk modulus of the water K_w:

$$\frac{1}{Q} = \frac{\alpha - n}{K_s} + \frac{n}{K_w} \tag{4a}$$

$$\alpha = \frac{1}{3}\mathbf{I} : \boldsymbol{\alpha} = \frac{1}{3}\boldsymbol{\alpha} : \mathbf{I} = 1 - \frac{K_T}{K_s} \tag{4b}$$

Governing equations (1) are written in dyadic notation valid in any orthogonal curvilinear coordinate system. Therefore, Eqs. (1) are valid not only in a rectangular coordinate system but also in any orthogonal curvilinear coordinate system such as a cylindrical coordinate system. With appropriate definitions for gradient and divergence, Eqs. (1) can be rewritten according to the chosen coordinate system. Detailed expressions in the cylindrical coordinate system, which are adopted in this study, are given in Malvern [16].

If initial strains are not taken into account, the total stress $\boldsymbol{\sigma}$ can be written in terms of the effective stress $\boldsymbol{\sigma}''$ of the solid skeleton, the pore pressure p, and the generalized Biot's constant $\boldsymbol{\alpha}$ as follows:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}'' - \boldsymbol{\alpha}p = \mathbf{D}[\boldsymbol{\varepsilon}] - \boldsymbol{\alpha}p = \mathbf{D}\left[\tfrac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)\right] - \boldsymbol{\alpha}p \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the strain of the solid skeleton.

On the boundary $\partial\Omega = \Gamma$ of domain Ω, the boundary conditions for the solid can be prescribed as follows:

$$\boldsymbol{\sigma}\mathbf{v} = \tilde{\boldsymbol{\sigma}}_n \ \text{ on } \ \Gamma = \Gamma_t \tag{6a}$$

$$\mathbf{u} = \tilde{\mathbf{u}} \ \text{ on } \ \Gamma = \Gamma_u \tag{6b}$$

where $\mathbf{v}$ denotes the outward unit normal vector on boundary Γ, and traction and displacement are prescribed on Γ_t and Γ_u, respectively. $\Gamma = \Gamma_t \cup \Gamma_u$ and $\Gamma_t \cap \Gamma_u = \phi$. Boundary conditions for the fluid may be written on boundary Γ, as follows:

$$p = \tilde{p} \ \text{ on } \ \Gamma = \Gamma_p \tag{7a}$$

$$\mathbf{v}\cdot\dot{\mathbf{w}} = \tilde{w}_n \ \text{ on } \ \Gamma = \Gamma_w \tag{7b}$$

where pressure and normal outflow are prescribed on Γ_p and Γ_w, respectively. $\Gamma = \Gamma_p \cup \Gamma_w$ and $\Gamma_p \cap \Gamma_w = \phi$.

The governing equations (1) can be reformulated in various ways depending on the chosen solution procedure. If the formulation is used in the original form without any modification, it is called the full mixed formulation or u–p–U formulation