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A simplified method for unified buckling and free vibration analysis of pile-supported structures in seismically liquefiable soils

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ABSTRACT

In seismic-prone zones with liquefiable deposit piles are routinely used to support structures (buildings/bridges). In this paper, a unified buckling and dynamic approach is taken to characterize this vibration. The pile–soil system is modelled as Euler–Bernoulli beam resting against an elastic support with axial load and a pile head mass with rotary inertia. The emphasis here is to obtain a simple expression that can be used by practicing engineers to obtain the fundamental frequency of the structure–pile–soil system. An approximate method based on an equivalent single-degree-of-freedom model has been proposed. Natural frequencies obtained from the exact analytical method are compared with approximate results. Proposed expressions are general as they are functions of non-dimensional parameters. It is shown that this simplified method captures the essential design features such as: (a) the continuous reduction of the first natural frequency of the structure–pile–soil system due to progressive reduction of soil stiffness due to liquefaction; (b) the reduction in the axial load-carrying capacity of the pile due to instability caused by liquefaction. The results derived in this paper have the potential to be directly applied in practice due to their simple yet general nature. An example problem has been taken to demonstrate the application of the method.

1. Introduction

1.1. *An overview on the collapse of pile-supported structures*

Collapse and/or severe damage of pile-supported structures are still observed after strong earthquakes despite the fact that a large factor of safety against axial capacity and bending due to lateral loads is employed in their design. Fig. 1(a) shows a pile-supported structure following the 1995 Kobe earthquake. Fig. 1(b), on the other hand, shows a schematic diagram of the same building along with the location of the cracks in the pile. All design codes employ a large margin of safety against the hinge formation (using partial factors), yet occurrences of pile failure due to liquefaction are abundant. This is strong evidence that there are perhaps other mechanisms governing these failures, which the code does not consider. A critical review of the current theories of pile failure and the hypothesis behind the current codes of practice can be found in [10].

1.2. *Bending mechanism due to kinematic loads on the pile and inertia of the superstructure*

The current understanding of pile failure as hypothesised by some codes of practice is as follows: in an earthquake if loose sands are saturated, they lose strength as excess pore water pressure is generated and the soil tends to liquefy. This means that if the soil is on a slope, it will flow downslope, which is often termed as *lateral spreading*. Up to now it has been assumed that the failure of these buildings was caused by the lateral pressure of the flow of the liquefied sand and any non-liquefied stabilised crust resting on the top of the liquefied soil (see for example [1–3,8,18,19,21–25,27,28,40,41,45,47]). Fig. 2 explains the hypothesis of failure. This mechanism is therefore based on kinematic bending failure; see [28]. The movement of the superstructure i.e. inertia force can also induce bending moments in the pile. The effects of inertia of the superstructure on the pile stresses are considered separately in [28]. They are not combined with the kinematic bending moments and the explanation can be found in [27]. Eurocode 8 advises designers to design piles against bending due to inertia and kinematic forces arising from the deformation of the surrounding soil. Other codes, such as NEHRP code and Indian Code [IS 1893, 2002] also focus on bending strength of the pile. In summary, the current understanding of pile failure simply

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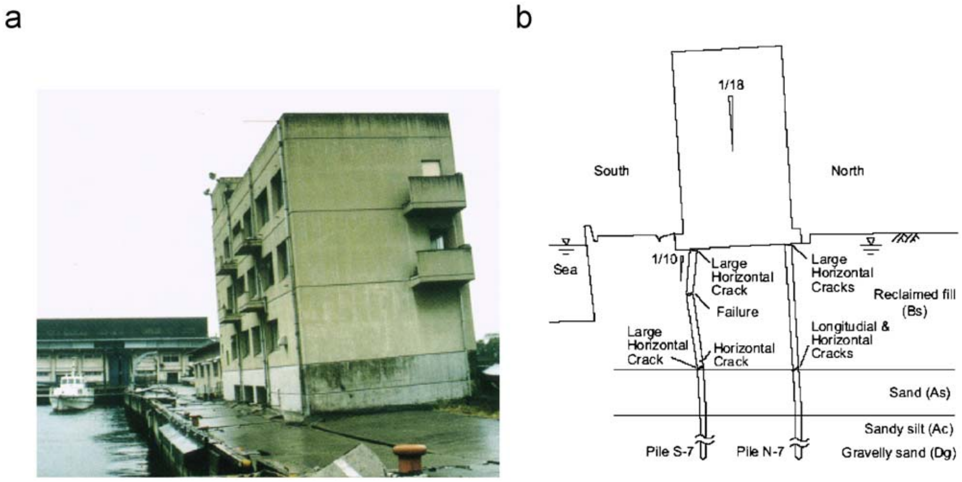

Fig. 1. (a) Tilting of a pile-supported building following the 1995 Kobe earthquake; (b) formation of crack. Photo courtesy K. Tokimatsu and Ref. [14].

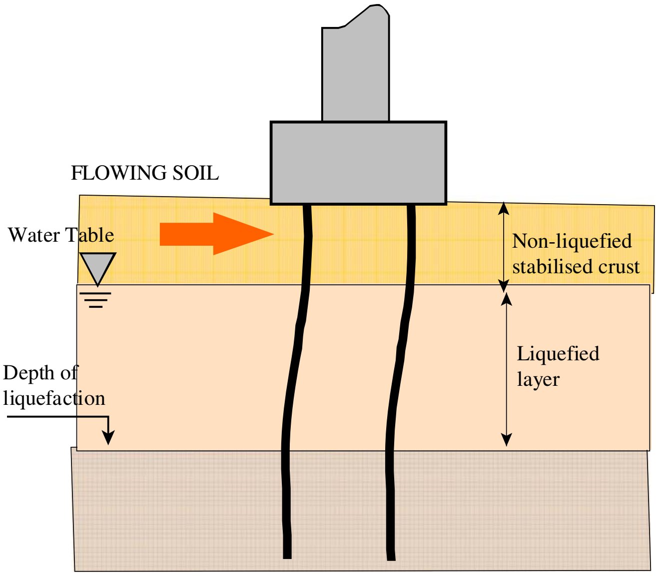

Fig. 2. Current understanding of pile failure.

treats piles as beams and assumes that the lateral loads due to inertia and soil movement cause bending failure of the pile. The stability issue due to the axial loads acting on the pile at all times and dynamic considerations are not taken into account.

1.3. *Buckling mechanism arising due to unsupported length of the pile in liquefiable zone*

A recent investigation by Bhattacharya et al. [11–13] conclusively showed that a pile becomes laterally unsupported in the liquefiable zone during strong shaking that led to another failure mechanism. The soil around the pile liquefies and loses much of its stiffness and strength, so the piles now act as unsupported long slender columns and simply buckle under the action of the vertical superstructure (building) loads. Therefore, this hypothesis is based on a buckling mechanism that has later been verified by other researchers; see for example [29–31,46,48]. Bhattacharya et al. [12] analysed 14 case histories of pile foundation performance based on buckling parameters. Though buckling mechanism can classify these pile failures, the location of hinge formation/cracks in the piles as observed in field survey cannot be explained by buckling instability theory. Criticisms of buckling mechanism can be found in [44]. This led to the search of any other mechanism of failure.

1.4. *Unified buckling mechanism and dynamic (resonance-type) failure*

Structurally, buckling of a slender column can be viewed as a complete loss of lateral stiffness to resist deformation. It is commonly known as an instability phenomenon. During liquefaction, if a pile buckles it can be concluded that the lateral stiffness of the pile is lost. From a dynamics point of view, as the applied axial load approaches the buckling load it can also be observed that the fundamental natural frequency of the system drops to zero [39]. Essentially, at the point where the natural frequency drops to zero, the inertial actions on the system no longer contribute. Thus, the system's dynamical equations of motion degenerate into a statics stability problem. During seismic liquefaction, the axial load on the pile in the liquefied zone increases due to the loss of shaft resistance. Due to this extra axial load, the stiffness of the pile–soil system reduces and so does the vibration frequencies. At the point of instability the fundamental vibration mode and buckling mode shapes are identical. Thus, as the soil transforms from solid to a fluid-like material i.e. from partial-liquefaction stage to full-liquefaction stage, the modal frequencies and shapes of the pile change.

Considering the first natural frequency of the pile–soil–superstructure system, it is suggested that the "other mechanism" may probably be the two effects arising from the removal of the lateral support the soil offers to the pile while in liquefied state. They are: (a) increase in axial load in the pile in the potentially unsupported zone due to loss of shaft resistance; (b) dynamics of pile-supported structure due to frequency-dependent force arising from the shaking of the bedrock and the surrounding soil than can cause dynamic amplification of pile head displacements leading to resonance-type failure.

1.5. *Winkler models for liquefied soil*

Beam on Non-linear Winkler foundation (BNWF) or "p–y" method is commonly used to analyse piles [7,17]. In "p–y" method, the soil is modelled as non-linear springs where 'p' refers to the lateral soil pressure per unit length of pile and the 'y' refers to the