Soil Dynamics and Earthquake Engineering 29 (2009) 517–524

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Determination of dispersive phase velocities by complex seismic trace analysis of surface waves (CASW)

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ARTICLE INFO

Article history:
Received 31 July 2007
Received in revised form
30 April 2008
Accepted 27 May 2008

Keywords:
Surface waves
Complex trace
Dispersion curve

ABSTRACT

A method for deriving experimental dispersion curves of surface waves from active source recordings is presented. The method is based on the complex seismic trace analysis of surface waves (CASW) and is applicable when only two receivers are available.

Reliable phase velocities are obtained when keeping the geophone interval smaller than one λ, allowing both a velocity structure as local as possible to be derived and to avoid long geophone spreads which are difficult to handle in urban areas. A large number of velocity estimates for each frequency can be estimated, even when using only two sensors, allowing statistical validation of the results, and providing a statistically defined uncertainty interval to be used in the dispersion curve inversion. The method is tested using synthetic seismograms and applied to real-world data, showing that it provides reliable estimates of apparent phase velocities. Although a final conclusion cannot yet be drawn, its application to observed data suggests it has the potential to be a useful method for distinguishing different modes.

1. Introduction

In recent years, methods based on the analysis of the dispersive properties of surface waves have found an increasing application in geophysical and geotechnical engineering site investigations. In particular, studies related to the assessment of the local amplification of ground motion following an earthquake require detailed knowledge of the S-wave velocity structure below a site. Standard invasive geotechnical methods that require the drilling of boreholes are quite expensive and therefore cannot be used to cover large urban areas. By contrast, non-invasive methods based on the analysis of the dispersive characteristic of surface waves are relatively low cost and allow large areas to be covered, while still attaining a reasonable depth of investigation. Such methods can be divided into two categories, depending upon the signal source required. Active source methods are those that require explosives, vibrators or drop weights, include spectral analysis of surface waves (SASW) [1] and multichannel analysis of surface waves (MASW) [2], while passive source methods, that is those that require the use of seismic noise, include spatial autocorrelation (SPAC) [3] and refraction microtremors (ReMi) [4].

Recently, many efforts have also been made in improving the inversion procedure by considering non-linear inversion schemes [5–7] and combining the inversion of the fundamental and higher modes [8,5,9–11].

The SASW method suffers mainly from the problem of error propagation during the phase-unwrap procedure [12,13], while MASW allows one to solve such problems, but requires a large number of geophones.

In this paper we proposed an alternative and rapid method for estimating the phase velocity of surface waves using active source recordings. The method, based on complex trace analysis [14,15] also works when only two receivers are available and provides reliable phase velocities within geophone intervals of one λ, allowing both the derived velocity structure to be as local as possible and to avoid long geophones spreads, which are difficult to handle in urban areas. A large number of velocity estimates for each frequency can still be found when using only two sensors, allowing the statistical validation of the results.

We show the effectiveness of the method using synthetic seismograms and an application to real data collected during an experiment in the Bonn area (Germany).

2. Method

The phase velocity of surface waves generated by an active source and traveling between two sensors can be obtained after calculating for the original seismograms their corresponding

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doi:10.1016/j.soildyn.2008.05.008

complex analytic traces. In this paper only vertical recordings, and therefore Rayleigh waves, are considered, but similar considerations can be made for the transverse components and hence, for Love waves.

2.1. Basic definitions

The complex seismic trace [14,15] is given by

$$c(t) = r(t) + \mathrm{i}q(t) \tag{1}$$

where $r(t)$ is the real seismic trace, i is the square root of -1 and $q(t)$ is the quadrature trace obtained as the Hilbert transform of the real trace.

The complex trace is obtained from the real trace by the following steps:

(1) the trace is zero-padded to a power of two sample length greater than or equal to twice the original length;
(2) transforming the real trace by Fast Fourier Transform (FFT), zeroing the amplitudes of negative frequencies and doubling the amplitudes for positive frequencies; and
(3) applying an inverse FFT.

The instantaneous amplitude and phase of the trace are given by

$$A(t) = [r^2(t) + q^2(t)]^{1/2} \tag{2}$$

and

$$\theta(t) = \arctan\,[q(t)/r(t)] \tag{3}$$

It is worth noting the time dependence of both parameters and, therefore, the retention of local significance.

2.2. Phase-velocity calculation: complex seismic trace analysis of surface waves (CASW)

In order to obtain the phase velocity at a certain frequency, the traces recorded at two different sensors are each first filtered using a Gaussian filter [16–18]

$$H(\omega, \omega_0) = \mathrm{e}^{-[(\omega-\omega_0)/(\alpha\omega_0)]2} \tag{4}$$

where ω_0 is the central frequency of the filter, α is the relative bandwidth and ω the frequency. Filtering is performed by first computing the Fourier transform of the trace, then multiplying the obtained complex spectrum by the Gaussian filter and then calculating the inverse Fourier transform of the filtered complex spectrum.

Successively, for each of the two filtered traces, the complex analytic trace is calculated.

A complex two-component trace may be defined from Eq. (1):

$$\boldsymbol{c}(t) = \boldsymbol{r}(t) + \mathrm{i}\boldsymbol{q}(t) \tag{5}$$

where the real trace $\boldsymbol{r}(t)$ is now a vector quantity defined by the two filtered traces, and each component of the quadrature trace $\boldsymbol{q}(t)$, is obtained by the application of the Hilbert transform to the corresponding component of $\boldsymbol{r}(t)$.

The geometric mean $G(t)$ of the instantaneous amplitude is given by

$$G(t) = \{[r_1^2(t) + q_1^2(t)][r_2^2(t) + q_2^2(t)]\}^{1/4} \tag{6}$$

where the subscripts 1 and 2 refer to the traces from each sensor.

The instantaneous phase difference $\phi(t) = \theta_1(t) - \theta_2(t)$, where $\theta_1(t)$ and $\theta_2(t)$ are the instantaneous phases, is computed between the two complex traces of the two-component trace by

$$\phi(t) = \arctan\left[\frac{r_2(t)q_1(t) - r_1(t)q_2(t)}{r_1(t)r_2(t) + q_1(t)q_2(t)}\right] \tag{7}$$

Eq. (7) is used to avoid correcting for phase unwrapping. Assuming that trace two is the one recorded at the larger distance from the source, we expect $\phi(t)$ to be positive.

Phase differences are retained only if calculated at a time t when $G(t)$ is greater or equal to 0.7 of the maximum, that is, only in the most energetic part of the seismogram. This reduces the influence of noise on the results, and allows us to calculate the local phase velocity

$$v(t) = (Df2\pi)/\phi(t) \tag{8}$$

where D is the distance between sensors and f is the frequency. It is worth noting the dependence of v on time and, therefore, the retention of local significance.

In the event of more than two traces being available, the procedure can be repeated by considering all possible combination of recordings.

Using this method, only phase differences corresponding to a maximum of one cycle of a given frequency can be correctly retrieved. This limits the maximum geophone interval $D_{\max}$ to a wavelength λ.

The minimum geophone interval $D_{\min}$ that provides well-constrained velocities can be chosen to be

$$D_{\min} = 10\Delta t v \tag{9}$$

where Δt is the sampling rate and v is the phase velocity. This equation shows that $D_{\min}$ is chosen here arbitrarily as the distance that allows us to sample at least 10 times the time difference of the wave arrivals at the two geophones. An uncertainty of one sample will therefore have a limited influence on the reliability of the estimated phase velocity at larger distances. However, $D_{\min}$ can be much smaller than what is generally required when using the standard SASW technique [19].

In order to ensure that the most energetic part of the seismograms correspond to surface waves and to avoid body wave effects [20], the minimum offset (source–receiver) should be large, that is being at least equal to or greater than the wavelength, λ of the lowest frequency of interest.

For each considered frequency, the $v(t)$ obtained for all combinations of available recordings are plotted and after a rapid visual inspection, the ones obtained from traces recorded at geophone intervals between $D_{\min}$ and $D_{\max}$, are selected and presented as a hystogram.

The phase velocity of a certain frequency can then be defined by either using statistical criteria (e.g. mean or median value of the distribution) or by simply picking the value that occurs most frequently (mode).

The procedure described above is summarized in Fig. 1.

3. A synthetic seismogram application

In order to test the proposed method, synthetic seismograms were calculated using a semi-analytical method that consists of an improved Thompson–Haskell propagator matrix algorithm that overcomes numerical instabilities by an orthonormalization technique [21].

Synthetic seismograms were generated considering the simple four-layer model described in Table 1. A source consisting of a single vertical force was located at the surface and the minimum offset to the geophones was set to 50 m in order to avoid both interference from body waves and to satisfy the plane wave assumption [19]. The maximum offset was fixed to 69 m, with a geophone interval of 1 m. The sampling rate was fixed to 1000 samples per second. The resulting