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Estimation of shallow subsurface shear-wave velocity by inverting fundamental and higher-mode Rayleigh waves

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Abstract

Shear (S)-wave velocity measurement of a shallow subsurface with a velocity reversal (e.g., roadbed soils) is challenging. The present study first evaluates the effects of the inclusion of different levels of noise in the fundamental mode phase velocities on the inverted models. Advantages of incorporating higher modes (up to the third mode) into the inversion process are then investigated. Our modeling results demonstrate that the ability to reveal low-velocity layers is degraded with increasing noise. An inversion performed with only fundamental mode data with a high degree of error may yield an unrealistic model. They also indicate that higher modes are relatively more sensitive to changes in S-wave velocity than is the fundamental mode. A better-behaved model can be obtained when the second mode data are inverted simultaneously with the fundamental mode data. A closer match with the true model can be achieved by simultaneously inverting three modes of phase velocities. We verify our modeling results with a real-world example from a roadbed site in Henan, China.

Keywords: Rayleigh waves; Higher modes; Dispersion curves; Shear wave velocity

1. Introduction

Accurate measurement of the S-wave velocity in such complex structures as roadbed soils is crucial to ensure highway quality, especially for the thin layer with a velocity reversal (a lower S-wave velocity layer between higher S-wave velocity layers), which usually leads to serious subsidence and deformation of highways [1,2]. In recent years, Rayleigh waves have been used increasingly as a noninvasive tool to invert S-wave velocity of near-surface materials for environmental and engineering applications [3,4]. Utilization of surface wave dispersive properties may be roughly divided into two methods. The methods are the spectral analysis of surface waves (SASW) [5–7], and the multi-channel analysis of surface waves (MASW) [8,9]. They have been widely and effectively applied to many geotechnical engineering site investigations [10–12]. Construction of a reliable S-wave velocity profile (S-wave velocity versus depth) through the analysis of planar, fundamental-mode Rayleigh waves is one of the most common ways for these two methods to exploit the dispersive properties of surface waves [13]. Comparisons to direct borehole measurements are generally good, but discrepancies do arise in some complex structures, especially for the thin layer with a velocity reversal [14].

A couple of reasons are likely responsible for discrepancies. The first is a stiff soil layer overlying a softer soil layer much easier induces a higher mode or multiple modes [15,16]. In such situation, higher modes likely take more energy than the fundamental-mode does in a higher frequency range. Shorter wavelength components of fundamental mode Rayleigh waves are obscured by these higher frequency data where higher modes of Rayleigh waves dominate, causing larger errors in fundamental mode data estimated in a higher frequency range [17]. The second reason is that picked fundamental mode phase velocities inevitably contained larger errors due to poorly constrained dispersion curves with a wider energy band at lower frequency ranges in heterogeneous roadbed soils.

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The third reason is that fundamental mode phase velocities are not sensitive to changes in the fine S-wave velocity structure because of limitations of resolution [17,18]. In addition, surface wave dispersion data obtained in field surveys are inherently noisy [19]. All of these factors result in the fact that an inversion performed with only fundamental-mode phase velocities fails to correctly interpret such complex geologic structures as roadbed soils.

In this paper, we first evaluate the effects of the inclusion of different levels of noise in data on the inverted models. Then, we demonstrate the necessity of fully utilizing multimodal phase velocities in the inversion of surface wave data when the thin layer with a velocity reversal is present. Finally, we verify our theoretical results using a real world example from a roadbed survey in Henan, China.

2. Inversion algorithm

Like many other geophysical inverse problems, the inversion of Rayleigh wave dispersion curves is a typical nonlinear inversion problem. Some local-search algorithms for inverting surface wave data have been presented by Xia et al. [9] and Lai et al. [20]. They are all stable and efficient for parameter fitting and have been used extensively to interpret surface wave data. We can use any of those two algorithms to perform inversion of multimode Rayleigh wave dispersion curves. In the current study, Occam's algorithm as developed by Lai et al. [20] was employed to invert the fundamental and higher mode Rayeigh wave data shown in this paper. The inversion strategy of this algorithm is to find the smoothest S-wave velocity profile subject to the constraint of a specified misfit between the observed and the calculated dispersion data.

Implementation of the Occam's algorithm requires a definition of smoothness or its converse roughness of a candidate solution that in our case is the $\mathbf{V}_\mathrm{S}$ profile. In practice, roughness may be defined by the following expression [21]:

$$R_1 = (\delta \mathbf{V}_\mathrm{S})^\mathrm{T}(\delta \mathbf{V}_\mathrm{S}), \quad (1)$$

where δ is an $n \times n$ matrix defining the *two-point-central* finite difference operator, the symbol $(\cdot)^\mathrm{T}$ indicates the transpose of an matrix, and n is the number of layers. The weighted root-mean-square (RMS) errors between the measured and the predicted phase velocities may be written as follows:

$$\varepsilon = \left\| \mathbf{W} \cdot (\mathbf{V}_\mathrm{R}^\mathrm{exp} - \mathbf{V}_\mathrm{R}^\mathrm{theo}) \right\| / \sqrt{m}, \quad (2)$$

where $\mathbf{V}_\mathrm{R}^\mathrm{exp}$ is an $m \times 1$ vector of measured Rayleigh wave phase velocities, $\mathbf{V}_\mathrm{R}^\mathrm{theo}$is an $m \times 1$ vector of theoretical Rayleigh wave phase velocities, $\|\cdot\|$ denotes the *Euclidean* norm of a vector, m is the number of phase velocities and $\mathbf{W}$ is a diagonal $m \times m$ weight matrix [22]:

$$\mathbf{W} = \mathrm{diag}\{1/\sigma_1, 1/\sigma_2, \ldots, 1/\sigma_m\}, \quad (3)$$

where σ_j $(j = 1,2,\ldots,m)$ is the standard deviation of measured Rayleigh wave phase velocities, estimated by repeated field tests or through a noise model.

Since S-wave velocities are the dominant influence on a dispersion curve [9], only S-wave velocities are sought in the current study while all other elastic parameters (namely P-wave velocity, density) and layer thickness are fixed to their known values. The solution of the nonlinear Rayleigh inverse problem consists of finding a vector $\mathbf{V}_\mathrm{S}$ that minimizes R_1 with the constraint that the residual error ε be equal to a specified value. The method of Lagrange multipliers is employed to solve this constrained optimization resulting in [22]

$$\mathbf{V}_\mathrm{S} = [\mu\delta^\mathrm{T}\delta + (\mathbf{W}\mathbf{J}_1)^\mathrm{T}\mathbf{W}\mathbf{J}_1]^{-1}(\mathbf{W}\mathbf{J}_1)^\mathrm{T}\mathbf{W}[\mathbf{J}_1\mathbf{V}_{\mathrm{S}0} + \mathbf{V}_\mathrm{R} - \mathbf{V}_{\mathrm{R}0}], \quad (4)$$

where μ is the Lagrange multiplier, which may be interpreted as a smoothing parameter, and the term $\mathbf{J}_1$ is the $m \times n$ Jacobian matrix whose elements are the partial derivatives of the multimode Rayleigh wave phase velocities with respect to S-wave velocities of the layers. Forward modeling of dispersion curves is based on Knopoff's method [23].

Eq. (4) is used iteratively to refine the estimated S-wave velocity profile until convergence. In our iterative procedure, an initial S-wave velocity profile with a uniform constant-velocity half-space is determined using the following formulas:

$$\mathbf{V}_{\mathrm{S}0} = (\mathbf{V}_\mathrm{R}^\mathrm{max} + \mathbf{V}_\mathrm{R}^\mathrm{min})/2\beta \quad \text{(for all layers)}, \quad (5)$$

where β is a constant ranging from 0.874 to 0.955 for Poisson's ratio ranges from 0.0 to 0.5 [9]. Based on our modeling results, β is chosen as 0.94. Inversion of the fundamental and higher mode data simultaneously has nothing special in terms of inversion algorithms except for including higher mode data in the inversion process as an extra set of data with equal weighting or different weighting dependent on data accuracy [17].

3. Modeling results

3.1. Possible effects of inclusion of different levels of noise in data on inversion results

Although we can accurately reconstruct complex subsurface models using error-free data, surface wave data acquired in field surveys are inherently noisy. In order to evaluate possible effects of the inclusion of different levels of noise in the fundamental mode dispersion data on inversion results, a complex earth model (Table 1) that contains a 1-m-thick thin layer with a velocity reversal between two higher S-wave velocity layers was used. This model was designed to simulate situations commonly encountered in shallow engineering site investigations.

In the current analysis, a frequency range of 5–100 Hz was used. Three contaminated data sets (open circles in Figs. 1a, b and 2a) were generated for comparison by