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Predicted tyre–soil interface area and vertical stress distribution based on loading characteristics

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ABSTRACT

The upper boundary condition for all models simulating stress patterns throughout the soil profile is the stress distribution at the tyre–soil interface. The so-called FRIDA model (Schjønning et al., 2008. Biosyst. Eng. 99, 119–133) treats the contact area as a superellipse and has been shown to accurately describe a range of observed vertical stress distributions. Previous research has indicated that such distributions may be predicted from tyre and loading characteristics. The objective of this study was to establish a stepwise calculation procedure enabling accurate predictions from readily available data. We used multiple regression to identify equations for predicting the FRIDA model parameters from measured loading characteristics including tyre carcass volume (V_T), wheel load (F_W), tyre deflection (L), and an expression of tyre inflation pressure (K_r) calculated as the natural logarithm of the actual to recommended inflation pressure ratio. We found that V_T and K_r accounted for nearly all variation in the data with respect to the contact area. The contact area width was accurately described by a combination of tyre width and K_r, while the superellipse squareness parameter, n, diminished slightly with increasing K_r. Estimated values of the contact area length related to observed data with a standard deviation of about 0.06 m. A difference between traction and implement tyres called for separate prediction equations, especially for the contact area. The FRIDA parameters α and β, reflecting the tyre's ability to distribute the stress in the driving direction and in the transversal direction, respectively, increased with increases in the relevant contact area dimension (length or width). The α-parameter was further affected by F_W, while K_r and L added to model performance for the β-parameter. The prediction accuracy of our models was tested on an independent data set and through a range of case studies. We found satisfactory small root mean square errors and effectively no bias in the comparisons. Further studies are needed, though, to quantify effects of topsoil consistencies deviating from those tested in this study.

1. Introduction

Soil compaction is a serious threat to soil functions and services (*e.g.*, Andersen et al., 2013; Batey, 2009; Håkansson and Reeder, 1994; Nawaz et al., 2013). Recent research has documented that compaction of subsoil layers persists at least for decades even in climates with frequent frost–thaw and wet–dry cycles (Berisso et al., 2012, 2013; Schjønning et al., 2013). Protective measures for avoiding soil compaction require tools and models for estimating the stresses applied to the soil surface by agricultural machinery. A first and basic prerequisite for modelling stress transmission in the soil profile is a quantitative knowledge of the stress distribution in the contact area between tyre and soil. Keller et al. (2014) documented the importance of using realistic rather than theoretical contact area stress distributions for accurate prediction of stress propagation throughout the soil profile. A range of soil compaction models nevertheless make use of very simple equations for the form and size of the tyre–soil contact area as well as the distribution of vertical stresses within this contact area.

Neither a uniform stress distribution nor those that can be described by simple power-law functions come close to the multitude of different distributions observed below real tyres (*e.g.*, Keller and Arvidsson, 2004; Keller et al., 2014; Schjønning et al., 2012). Keller (2005) combined a power-law function and a decay

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function to devise a mathematical tool that would reasonably fit most distributions of stresses in the tyre–soil contact area. In short, the Keller (2005) model includes two form parameters, α and δ, which determine the stress distribution, respectively, in and across the driving direction. The Keller (2005) model was slightly modified by Schjønning et al. (2008) by normalizing the form parameter δ by the width of the tyre contact area. The modified model by Schjønning et al. (2008) was named FRIDA, and the tyre-width independent form parameter was called β. Keller (2005) used his stress-distribution parameter in the driving direction, α, as a kind of fitting factor in order to match the integrated vertical stresses with the wheel load applied. In the modified FRIDA model (Schjønning et al., 2008) this matching is done by including a separate factor. The benefits of these modifications are that α and β both become 'universal' form parameters that do not include information of tyre dimensions and wheel load. This, in turn, improves the comparison of stress distributions across differently sized tyres.

The FRIDA model and its Keller (2005) predecessor both use a superellipse to describe the shape of the tyre–soil contact area. This is based on a suggestion by Hallonborg (1996). The superellipse model has proven ideal for describing the contact area across a wide range of tyres, wheel loads and inflation pressures (*e.g.*, Lamandé and Schjønning, 2008). The superellipse is described by three parameters, a, b, and n, where a and b are half the length of the minor and major axes in the superellipse, and n is the 'squareness', for tyres typically taking values between 2 (an ellipse) and 9 (a rectangular-like super-ellipse) (Schjønning et al., 2006).

We hypothesize that the size and shape of the tyre–soil contact area as well as the distribution of vertical stress expressed through the FRIDA model parameters α and β are determined by and thus may be predicted by the loading characteristics. These include the tyre dimensions, the inflation pressure, and the load applied to the wheel. The purpose of this study was to test the above hypothesis and to establish a scheme for stepwise calculations for such predictions.

2. The FRIDA model

The FRIDA model (Keller, 2005; Schjønning et al., 2008) may in short be described as follows. The periphery of the tyre–soil contact area was modelled by a superellipse (Hallonborg, 1996), which in an orthogonal coordinate system with centre at the origin is given by:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1, \quad (1)$$

where a and b are half the width of the minor and major axes [m], and n is the "squareness". If

$$\Omega = \left\{(x,y)\left|\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n \le 1\right.\right\} \quad (2)$$

denotes the boundary and interior of the superellipse, the FRIDA model describes the distribution of the vertical stress, $\sigma(x,y)$, in the contact area:

$$\sigma(x,y) = F_W C(\alpha,\beta,a,b,n) f(x,y) g(x,y) \text{ for } (x,y) \in \Omega \text{ and 0 otherwise with} \quad (3)$$

$$f(x,y) = \left\{1 - \left|\frac{x}{l_x(y)}\right|^\alpha\right\} \quad (3a)$$

$$g(x,y) = \left\{\left(1 - \left|\frac{y}{w_y(x)}\right|\right)\left(\frac{1}{g_{max}}\right)\exp\left(-\beta\left(1 - \left|\frac{y}{w_y(x)}\right|\right)\right)\right\} \quad (3b)$$

where g_{max} is the maximum value of g in the range $(0 < y < w_y(x))$ expressed in terms of β:

$\beta \le 1$: $g_{max} = \exp(-\beta)$
$\beta > 1$: $g_{max} = \exp(-1)/\beta$

F_W is the wheel load in kN, $C(\alpha,\beta,a,b,n)$ is a function of the parameters α, β, a, b, and n, defining an integration constant to ensure that when integrating $\sigma(x,y)$ over the contact area Ω, the total load is F_W. Furthermore, $l_x(y)$ is half the length of the footprint in the x-direction at a given y-value, and $w_y(x)$ the half width in the y-direction at a given x-value. The a and b terms are thus identical to, respectively, $l_x(y)$ at $y = 0$ and $w_y(x)$ at $x = 0$. The f and g functions describe the shape of the stress distribution in the driving direction and in the direction perpendicular to the driving direction (across the wheel), respectively.

3. Materials and methods

3.1. Data sets

Two traction and five implement tyres were selected for tests on a sandy loam soil at the Research Centre Foulum, Denmark (56°30′N, 9°34′E) in 2005 (Table 1). The results were presented in an institutional report (Schjønning et al., 2006) and published internationally (Lamandé and Schjønning, 2008; Schjønning and Lamandé, 2010; Schjønning et al., 2008, 2012). Here we will give a short description of the tests.

The two traction tyres (Michelin Xeobib 650/60R38 and Kleber Topker 650/75R38) were tested at two wheel loads: ~30 and ~60 kN. For the Kleber Topker tyre, we used the inflation pressures recommended by the manufacturer for traffic in the field at low torque (<10 km h^{-1} driving speed). The Michelin Xeobib tyre has a recommended maximum wheel load of ~40 kN with a corresponding recommended inflation pressure of 100 kPa, independent of driving speed. However, in order to test the potential of the highly flexible Xeobib tyre at ratios of wheel load and inflation pressure approximately resembling those for other tyres, we used the inflation pressures listed in Table 1.

The five implement tyres were all low-lug agricultural tyres most often used on towed agricultural trailers (*e.g.*, slurry tankers): Euroband SA 385/65R22.5, Nokian ELS Radial 560/45R22.5, Trelleborg TWIN 4004 700/50-26.5, Michelin CargoXbib 650/65R30.5 and Nokian ELS Radial 800/50R34. The Euroband tyre is originally designed for trucks and busses. It exhibits a strong construction and is usually operated at high inflation pressure. The Trelleborg TWIN tyre is a so-called belted crossply tyre claimed to possess some of the characteristics of radial-ply tyres. The Nokian 560, Nokian 800 and the Michelin CargoXbib tyres are all radial-ply tyres that differ with respect to their size and aspect ratios. All implement tyres were tested at two wheel loads (30 and 60 kN) at rated inflation pressures. In addition, the Nokian 800 and the Michelin CargoXbib tyres were tested at a wheel load of ~83 kN, and at the 60 kN load at under-inflated and over-inflated pressures (Table 1). Throughout this paper, the tyres are labelled as follows (cf. Table 1): Xeobib, Kleber, Euroband, Nokian560, Twin, Cargoxbib and Nokian 800, for the two tractor and five implement tyres, respectively.

The Foulum soil is a sandy loam with a relatively high content of fine sand (0.02–0.2 mm; Table 2). The test field had been mouldboard-ploughed to ~0.2 m depth about seven months prior to the wheeling tests, which took place at a water content slightly lower than field capacity (Table 2), which corresponds approximately to a matric potential of −100 hPa (pF2; Schjønning and Rasmussen, 2000). The soil properties listed in Table 2 are