Soil & Tillage Research 141 (2014) 10–18

Contents lists available at ScienceDirect

Soil & Tillage Research

journal homepage: www.elsevier.com/locate/still

A semi-empirical traction prediction model for an agricultural tyre, based on the super ellipse shape of the contact surface

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ARTICLE INFO

Article history:
Received 26 November 2013
Received in revised form 21 February 2014
Accepted 17 March 2014

Keywords:
Tyre traction
Super ellipse
Traction model
Contact surface

ABSTRACT

This study presents a semi-empirical model for predicting the traction force and traction efficiency for a 2WD agricultural tractor, assuming that the shape of the tyre–ground contact area is a super ellipse. The model assumes that, under the vertical load, the wheel sinks into the soil and that the load induces tyre deflection; as a result, the virtual radius of the deformed tyre increases, as it flattens in the tyre–ground contact area. A computer programme was developed in order to solve the equations of the model, using an iterative process. In order to evaluate the maximum traction force it was assumed that it is limited only by the soil shear strength and the Mohr–Coulomb equation was used to calculate the soil maximum shear stress.

Because the available literature presents a large range of values for the super ellipse exponent, field tests were performed in order to validate the model; the experimental data were collected during ploughing tests. In order to evaluate the precision of the model the predicted data were compared with the test data by the means of a goodness-of-fit analysis.

The results predicted by the model show that the length of the tyre–ground contact surface is not affected by the shape of the contact surface (value of the super ellipse exponent), while increasing the value of the super ellipse exponent results in an increased of the area of the contact surface.

The best fit between model data and experimental data was achieved when the value of the super ellipse exponent was set to $k = 3.5$.

1. Introduction, literature review and objective

Tractors are the basic tool for performing many agricultural operations and the net traction force developed by the tractor is a key element for the appropriate matching and selection of the tractor-implement system. A model for predicting the traction performance of an agricultural tractor would be a useful tool for researchers and engineers in solving problems regarding the operating parameters of the tractor, soil compaction due to traffic etc. The results produced by such a model would allow the engineer to evaluate the consequences of changing the model input parameters; the effects of tractor and soil specific parameters could be studied without appealing to expensive and time consuming field tests. Consequently, many researches were aimed to provide a model for the accurate description of the tyre–soil interaction. According to Tiwari et al. (2010) the tyre–soil interaction models can be based on empirical, semi-empirical and analytical methods.

Empirical methods are mainly based on soil properties (cone index, plate sinkage, shear strength) using similitude and dimensional analysis. The empirical models were developed using traction data recorded from operating vehicles and, for some of them, cone index, measured with a standard cone penetrometer, was the only soil property taken into account. Wismer and Luth (1972) developed a widely used model for bias tyres, based on a soil–tyre numeric, which under predicted the traction force when applied to radial tyres. Clark (1985) modified the Wismer–Luth equation to fit data collected with a low power agricultural tractor and the coefficients involved were individually optimized to fit the model with experimental data. The Brixius (1987) equations, as a refinement of the Wismer and Luth equations, expressed the gross traction ratio (GTR) as a function of slip and wheel mobility number, using a curve fitting technique in order to evaluate the coefficients for the traction equation (Abbaspour-Gilandeh et al., 2007).

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http://dx.doi.org/10.1016/j.still.2014.03.007

In the semi-empirical models, the shear deformation of soil is considered; the models are based on soil parameters obtained by the means of a bevameter technique (penetration and shear tests), assuming that the vertical deformation of soil is similar to the deformation under a sinking plate, while the shear deformation of soil under a traction device is similar to the shear action of a torsion device (Tiwari et al., 2010). The parameters involved in the equations are determined experimentally. For agricultural soils, the Janosi and Hanamoto (1961) equation is one of the most frequently used.

The analytical models are formulated using elasticity and plasticity approaches. Elasticity models are based on the classical mechanical contact theory in order to predict deformations and stresses (using, for example, the Boussinesq's approach), while plasticity based models take into account material (soil) failure theories. Upadhyaya et al. (1990) concluded that analytical models never adequately describe the interaction between tyre and soil, as they have inconsistent physical properties, while traction models based on empirical relationships worked better than the analytical methods.

The shape of the contact area between tyre and soil is a key parameter for the development of the traction model. Many traction models assume that the tyre - soil contact patch is symmetrical; Grecenko (1995) suggested that it has an elliptical shape and its area could be obtained by multiplying the product of the length and width of the contact area by a coefficient with values between 0.8 and 0.9. Hallonborg (1996) used a super ellipse model for the tyre–ground contact area; the value of the positive exponent in the equation defined the shape of the contact patch. McKyes (1985) developed a simple formula in order to compute the area of the contact patch, using the diameter and width of the tyre. Schjønning et al. (2008) also took into account the super ellipse to describe the symmetrical shape of the contact patch, with values of the exponent comprised between 2.45 and 4.51, depending upon tyre inflation pressure and model. Saarilahti (2002) performed a comparison of the footprint area given by different models and concluded that, for the same tyre, the contact patch area given by different models had values comprised between 0.05 and 0.25 m^2. Shmulevich and Osetinsky (2003) developed a model for wheel soil interaction assuming that tyre–ground contact surface can be represented in a parabolic form in the longitudinal direction; the analysis of the goodness-of-fit between the model and the experimental data was based on the Pearson correlation coefficient. Keller (2005) also considered the contact patch as a super ellipse and made measurements of the vertical stress below tyres using compression cells. He developed an equation relating the shape exponent of the super ellipse to some characteristics of the tyre and remarked a good agreement with the model taken into account.

The main objective of this study was the development of a semi-empirical model for predicting the traction force and traction efficiency for a 2WD agricultural tractor, assuming that the shape of the tyre–ground contact area is a super ellipse. Because the available literature presents a large range of values for the super ellipse exponent, field tests were performed in order to validate the model; the experimental data were collected during ploughing tests. In order to evaluate the precision of the model the predicted data were compared with the test data by the means of a goodness-of-fit analysis.

2. Traction model

The traction model is based on the schematics shown in Fig. 1. The model assumes that, under the vertical load (G), the wheel sinks into the soil, reaching the depth (z_c) and the load induces tyre deflection (z_p); complete soil rebound behind the wheel was considered (elastic behaviour of soil). As a result, the radius of the contact patch becomes r_d ($r_d > r_0$), and the length of the contact patch is:

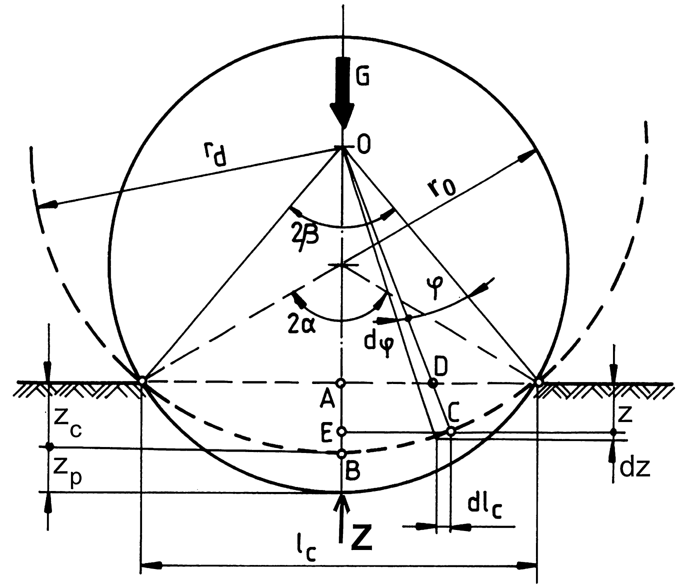

Fig. 1. Schematics of the wheel-soil interaction model.

$$l_c = 2 \cdot r_d \cdot \sin\beta = 2 \cdot r_0 \cdot \sin\alpha \tag{1}$$

From Fig. 1:

$$z = \bar{O}\bar{E} - \bar{O}\bar{A}, \tag{2}$$

and, finally:

$$z = r_d \cdot [\cos(\beta - \varphi) - \cos\beta]. \tag{3}$$

The pressure–sinkage relationship (Adams, 2002; Rashidi and Gholami, 2008) finally leads to:

$$G = \int_0^{2\beta} p \cdot b(\varphi) \cdot r_d \cdot \cos(\beta - \varphi) \cdot d\varphi = k \cdot \int_0^{2\beta} r_d^{n+1} \cdot [\cos(\beta - \varphi) - \cos\beta]^n \cdot b(\varphi) \cdot \cos(\beta - \varphi) \cdot d\varphi, \tag{4}$$

where φ is the current angle (defining the position along the contact surface, as shown in Figs. 1 and 2), p is the normal pressure [kPa], z is the soil deformation [m] and k [kPa/m^n] and n are constants.

The shape of the contact patch is assumed to be a super ellipse (Keller, 2005). In an orthogonal co-ordinate system (Fig. 2) the equation of the super ellipse is:

$$\left(\frac{2 \cdot x}{l_c}\right)^k + \left(\frac{2 \cdot y}{b}\right)^k = 1 \tag{5}$$

where k is the super ellipse exponent, the minor axis of the super ellipse is assumed to be equal to the tyre width b (Keller, 2005) and l_c is the major axis of the super ellipse, representing the length of contact area.

The equation finally leads to the width of the contact patch as a function of the current angle:

$$b(\varphi) = 2 \cdot y = \sqrt[k]{\frac{(l_c \cdot b)^k - [2 \cdot b \cdot r_d \cdot \sin(\beta - \varphi)]^k}{l_c^k}} \tag{6}$$