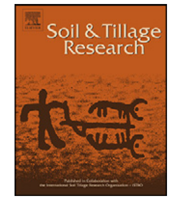
Soil & Tillage Research 125 (2012) 30–37

Contents lists available at SciVerse ScienceDirect

Soil & Tillage Research

journal homepage: www.elsevier.com/locate/still



The velocity of shear waves in unsaturated soil

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ARTICLE INFO

Keywords:
Soil
Shear wave velocity
Unsaturated
Consolidation
Tri-axial testing

ABSTRACT

The velocities of shear waves V_s in two soils, a loamy sand and a sandy clay loam, were measured at various matric potentials and confining pressures. We used a combination of Haines apparatus, pressure plate apparatus and a Bishop and Wesley tri-axial cell to obtain a range of saturation and consolidation states. We proposed a single effective stress variable based on a modification to Bishop's equation which could be used in a published empirical model (Santamarina et al., 2001) to relate shear wave velocity to soil physical conditions. Net stress required a nonlinear transformation. Matric potential was converted into suction stress with the function proposed by Khallili and Khabbaz (1998), thus requiring an estimate of the air entry potential. We found it was possible to fit V_s to void ratio, net stress and matric potential with a set of four parameters which were common to all soils at various states of saturation and consolidation. In addition to the data collected for this study we also used previously published data (Whalley et al., 2011). The utility of shear wave measurements to deduce soil physical properties is discussed.

1. Introduction

Previously we have explored how consolidation of saturated soil affects the velocity of shear waves (Whalley et al., 2011). We used an empirical model for shear wave velocity, V_s, described by Santamarina et al. (2001) and written as

$$V_s = AF_e\left(\frac{\sigma}{1\,\text{kPa}}\right)^{\beta} \tag{1}$$

where F_e is a the void ratio normalization factor which takes account of changes in elastic modulus (shear modulus in this case) due to differences in porosity, σ is the effective stress (often σ' is used but here we use σ to be consistent with Whalley et al. (2011)), A is the value of V_s when σ = 1 kPa and β is a fitted parameter. The value of the void ratio normalization factor, F_e is given as follows

$$F_e = \frac{(2.97 - e)^2}{1 + e} \tag{2}$$

where e is the void ratio (Lo Presti, 1995). The constant in the numerator bracket can be obtained empirically by fitting to data, but a value 2.97 has been recommended for angular particles (Santamarina et al., 2001) and has been found to be suitable for a range of agricultural soils (Whalley et al., 2011). In most agricultural soils void ratio is very sensitive to the effective stress and they are related to each other by the compression characteristic which is commonly written as

$$e = n - \lambda \log_{10} \sigma \tag{3}$$

where n and λ are fitted parameters and depend on soil type. By combining these equations we were able to collapse V_s data from a range of soils and consolidation states onto a common relationship (Whalley et al., 2011). The estimated value of β was 0.39 which is midway between the expected values of 0.25 for rough angular particles and 0.75 for a porous material where particle contacts are governed by Coulomb forces (see Santamarina et al., 2001).

A limitation of the common relationship, described by Whalley et al. (2011), is that it is restricted to saturated soil. Here the stress supported by the fabric of the soil is given by

$$\sigma = \sigma_c - u_w \tag{4}$$

where σ is the effective stress, σ_c is the total stress (in this case the pressure in the tri-axial cell) and u_w is the pressure of the soil water. This is the effective stress according to Terzaghi (see Santamarina et al. (2001) or Mitchell and Soga (2005) for discussion). For *unsaturated* soil the effective stress is given by Bishop and Blight (1963) as

$$\sigma = (\sigma_c - u_a) - \chi(u_w - u_a) \tag{5}$$

where $(u_w - u_a)$ is the matric potential $(\sigma_c - u_a)$ is the net stress (excess of total stress over air pressure), χ is a factor that converts matric potential ($\psi = u_w - u_a$) into what is sometimes referred to a "suction stress" (e.g. Lu et al., 2010) and u_a is the air pressure in the

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http://dx.doi.org/10.1016/j.still.2012.05.013

soil sample. Alonso et al. (2010) observed that because of a lack of data, the degree of saturation, S, is often used as a candidate for χ. In the special case of tensile failure testing Mullins (2000) found that for $S > 0.5$ the simple assumption that $\chi = S$ worked well. Essentially χ weights the contribution of the matric potential according to its effect on effective stress. Although, S is widely used to estimate χ (e.g. Whitmore et al., 2011) an alternative has been suggested by Khallili and Khabbaz (1998) who proposed that

$$\chi = \left(\frac{\psi}{\psi_{ae}}\right)^{-0.55} \quad \text{for } \psi < \psi_{ae} \text{ otherwise } \chi = 1 \tag{6}$$

where ψ_{ae} is the matric potential at which air invades a drying soil, often called the air entry potential (see Mitchell and Soga (2005) for discussion).

It is known that the velocity of elastic waves in unsaturated soil depends on both the net stress applied to the soil volume under test as well as the saturation state. This has been shown clearly by Lu and Sabatier (2009) in a two year survey of an outdoor field site exposed to the prevailing weather. They measured matric potential, water content and the velocity of compression waves (P waves), V_p. For the soil they monitored they were able to develop an empirical power law relationship between V_p and matric potential of sufficient quality ($r^2 = 0.92$) to form the basis of a calibration. They also found correlations between V_p and soil water content. The measured values of V_p were also sensitive to depth; deeper layers tended to have higher wave speeds (V_p). This observation was explained by the effect of overburden and it is consistent with the Bishop and Blight equation for effective stress. In deeper layers the higher overburden pressure would tend to increase wave speeds but these layers also tend to be wetter which reduces wave speed (see Velea et al., 2000).

This paper has two objectives. First, we wish determine the relationship between χ and soil moisture in unsaturated agricultural soils. Specifically we wanted to test the function for χ proposed by Khallili and Khabbaz (1998) which has never been applied to agricultural soils. Our second objective is to characterize the additive nature of net stress ($\sigma_c - u_a$) and suction stress ($\chi\psi$) in so far as they can be used to estimate a single effective stress variable for use in the empirical model of Santamarina et al. (2001) for the velocity a shear wave in variably consolidated and saturated soil. Here the effects of stress history will not be considered.

2. Materials and methods

2.1. Soils and other data

The properties of the agricultural soils used in this work are listed in Table 1. One of the soils (Butt Close) was previously used and described by Whalley et al. (2011). Warren Field soil has not been previously used to make V_s measurements and it was included here because it is from an important group of soils used for arable agriculture. We included the data of Whalley et al. (2011), but excluded any data collected when the soil was on the elastic rebound curve.

2.2. Measurement of V_s in unsaturated, but unconfined sands and soil

We used the Haines apparatus shown in Fig. 1 to measure the velocity of shear waves in sand at various matric potentials. To measure the shear wave velocity in two agricultural soils (see Table 1) we used a pressure plate apparatus (see Fig. 1).

2.3. Measurement of V_s in confined soil

Shear wave velocity in consolidated soils was measured using a Bishop and Wesley tri-axial cell (GDS instruments, 32 Murrel Green Business Park, Hook, Hampshire, RG27 9GR, UK) (see Bishop and Wesley (1975) and Fig. 1a). These soil samples, 100 mm long and 50 mm in diameter, were obtained by packing thin layers of soil into a split brass mould with an axial pressure of approximately 10 kPa. Before packing the water contents were adjusted so the soils were at their most compactable (see Gregory et al., 2010). Once in the triaxial cell, the soils were saturated by increasing the pressure of the water confining (σ_c) the soil sample and the pressure of the soil water u_w to 600 and 590 kPa respectively over a period of 24 h. The soils were then consolidated to a range of effective stresses between 10 (initial condition) and 400 kPa. During consolidation a record of the sample length (measured with a LVDT) was kept. The S wave was generated by a piezoelectric device in the top-cap and detected in the pedestal by a similar device. This apparatus is commercially available from GDS, who supplied the Bishop and Wesley cell. The input electrical signal was a single cycle of a sine wave with a 0.2 ms period. The time for the S wave to travel through the soil sample was determined by comparing the input and detected shear waves (Whalley et al., 2011).

For the loamy sand (Butt Close) but not on the sandy clay loam (Warren Field) measurements were made on both normally consolidated soil and also soil during elastic rebound and in both cases the applied effective stress was isotropic. On the Butt Close loamy sand the relationships between V_s and effectives stress was not greatly affected by stress history (Whalley et al., 2011) and the effects of rebound could be ignored. During the tests on Butt Close soil at an effective stress of 98 kPa, the soil was drained by simultaneously increasing the pore air pressure (u_a) and the confining pressure in the triaxial cell (σ_c). This had the effect of

Table 1
Description of soils used in this work (Butt Close and Warren Field) as well as those soils used by Whalley et al. (2011) for which data was used in the curve fitting.

Property	Units	Broadbalk FYM	Rowden	Butt Close	Warren
Location		Rothamsted Res.	North Wyke Res.	Woburn Farm Res.	Woburn Expt. Farm
		Hertfordshire	Devon	Bedfordshire	Bedfordshire
Grid reference	GB national grid	TL121134	SX652994		SP968364
	Latitude	51°48′36″N	50°46′42″N	52°00′42″N	52°01′06″N
	Longitude	00°22′30″W	03°54′54″W	00°32′42″W	00°35′30″W
Soil type	SSEW group	Paleo-argillic brown earth	Stagnogley soil	Brown Earth	Brown Earth
	SSEW series	Batcombe	Hallsworth	Cottenham	Flitwick
	FAO	Chromic Luvisol	Gleyic Luvisol	Cambic Arenosol	Dystric Cambisol
	USDA	Paleudalf	Haplaquept	Udipsamment	Hapludalf
Landuse		Arable; cereals; farmyard manure	Grass; unfertilized; grazed	Arable	Arable; cereals; beans
Sand (2000–63 μm)	$g\,g^{-1}$ dry soil	0.252	0.147	0.875	0.538
Silt (63–2 μm)	$g\,g^{-1}$ dry soil	0.497	0.396	0.055	0.203
Clay (<2 μm)	$g\,g^{-1}$ dry soil	0.252	0.457	0.072	0.260
Texture	SSEW class	Clay loam	Clay	Loamy sand	Sandy clay loam
Particle density	$g\,cm^{-3}$	2.508	2.439	2.65	2.587
Organic matter	$g\,g^{-1}$ dry soil	0.060	0.138	0.01	0.038