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Technical Paper

Energy-less strain in granular materials – Micromechanical background and modeling

Susumu Iai*, Kyohei Ueda

Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto 611-0011 Japan

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Abstract

During the macroscopic deformation of granular materials, which consist of an assembly of particles, the micromechanical structures change by forming new contacts or losing existing contacts. Among touching and non-touching particles in granular materials, touching particles contribute to macroscopic stress, which is given as the tensorial average of the contact forces between the particles. However, both touching and non-touching particles contribute to macroscopic strain, which is given by the tensorial average of the relative displacements between the particles. As non-touching particles lack contact force and contribute to no internal work, a constraint condition must be imposed on the macroscopic strain due to the non-touching particles; the work induced by the strain due to the non-touching particles must be zero. The strain that meets this constraint condition is called energy-less strain and it is studied further for applications to the macroscopic constitutive modeling of granular materials. The framework of the strain space multiple mechanism model is used to upscale the micromechanical structure of granular materials into the macroscopic constitutive modeling. Through this framework of study, the volumetric strain of the energy-less strain is identified as the dilative component of dilatancy in granular materials. The evolution of induced anisotropy, in terms of the fabric of the energy-less strain, is also identified.

Keywords: Anisotropy; Dilatancy; Fabric tensor; Granular material; Micromechanics; Strain energy

1. Introduction

The dilatancy of granular materials has been a central issue in soil mechanics. Well-known studies by Newland and Allely (1957), Rowe (1962), Shibata (1963), Roscoe et al. (1963), and Ohta and Hata (1971) were conducted around the 1960s. Of particular significance was the energy-based stress dilatancy relation for the original Cam–Clay model by Schofield and colleagues (Roscoe et al., 1963; Schofield and Wroth, 1968). The relation for the triaxial condition is written as

$$\bar{p}\mathrm{d}v_\mathrm{p} + \bar{q}\mathrm{d}\bar{\gamma}_\mathrm{p} = M\bar{p}\mathrm{d}\bar{\gamma}_\mathrm{p} \qquad (1)$$

where $\bar{p}$ is the effective mean stress $(=(\sigma'_\mathrm{a}+2\sigma'_\mathrm{r})/3)$ (compression positive), $\bar{q}$ is the deviator stress $(=\sigma'_\mathrm{a}-\sigma'_\mathrm{r})$, v_p is the plastic volumetric strain $(=(\varepsilon_\mathrm{p})_\mathrm{a}+2(\varepsilon_\mathrm{p})_\mathrm{r})$ (compression positive), and $\bar{\gamma}_\mathrm{p}$ is the plastic deviator strain $(=(2/3)\{(\varepsilon_\mathrm{p})_\mathrm{a}-(\varepsilon_\mathrm{p})_\mathrm{r}\})$. Subscripts a and r indicate the axial and the radial components, respectively.

The volumetric strain in Eq. (1) can be divided into the following two components:

$$\mathrm{d}v_\mathrm{p} = \mathrm{d}v_\mathrm{p}^\mathrm{c} + \mathrm{d}v_\mathrm{p}^\mathrm{d} \qquad (2)$$

*Corresponding author. Tel.: +81 774 38 4090; fax: +81 774 38 4094.
E-mail address: iai@geotech.dpri.kyoto-u.ac.jp (S. Iai).
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where

$$\mathrm{d}v_{\mathrm{p}}^{\mathrm{c}} = M\mathrm{d}\overline{\gamma}_{\mathrm{p}} \tag{3}$$

$$\overline{p}\mathrm{d}v_{\mathrm{p}}^{\mathrm{d}} + \overline{q}\mathrm{d}\overline{\gamma}_{\mathrm{p}} = 0 \tag{4}$$

Substituting Eqs. (2)–(4) into the left-hand side of Eq. (1) confirms that Eqs. (2)–(4) satisfy Eq. (1) and offer an alternative look at the energy-based stress dilatancy relation for the original Cam–Clay model. Volumetric strain $\mathrm{d}v_{\mathrm{p}}^{\mathrm{c}}$, in Eq. (3), describes the contractive component of dilatancy, whereas volumetric strain $\mathrm{d}v_{\mathrm{p}}^{\mathrm{d}}$, in Eq. (4), describes the dilative component of dilatancy.

As shown in Eq. (4), the strain $\left(\mathrm{d}v_{\mathrm{p}}^{\mathrm{d}}, \mathrm{d}\overline{\gamma}_{\mathrm{p}}\right)$, which includes the volumetric strain of the dilative component of dilatancy, does not induce work. Consequently, it represents the energy-less component of strain (hereafter called the energy-less strain) (Iai, 1994). In the 1940s, Taylor documented the importance of the energy-less strain component (Taylor, 1948). Imagining that the dilatation of sand is necessary for the release of shear deformation against interlocking, in order to save (or absorb) shear energy, he called the mechanism of this energy-less strain "interlocking".

The objective of this study is to elucidate the mechanism of energy-less strain in granular materials and to discuss the role of energy-less strain in constitutive modeling using the framework of the strain space multiple mechanism model, which relates macroscopic strain to macroscopic stress (Iai, 1993a; Iai and Ozutsumi, 2005; Iai et al., 2011). To facilitate the understanding of the strain space multiple mechanism model, as a framework for upscaling the micromechanical structure to the macroscopic mechanism, the discussion on this model is presented by referring to a previous study by Rothenburg and Bathurst (1989) on the assembly of plane circular particles.

2. Energy-less strain

The macroscopic effective stress in granular materials is given by the average over contact forces $\mathbf{P}$ within a representative volume element with volume V (Christoffersen et al., 1981; Mehrabadi et al., 1982) as

$$\boldsymbol{\sigma}' = \frac{1}{V}\sum_{\mathrm{tc}} l\mathbf{P} \otimes \mathbf{n} \tag{5}$$

where l denotes the length of the branch, $\mathbf{n}$ is the direction of the contact normal (or along the branch connecting the particle centers) (Fig. 1), and the symbol tc represents the contacts between touching particles. In two dimensions, the volume refers to the area multiplied by the unit thickness.

Unlike the tensorial average, used to explain the macroscopic stress for touching particles (Eq. (5)), the tensorial average, used to define the macroscopic strain, has contributions from both touching and non-touching particles. Thus, the main problem in defining strain for a granular assembly is how to determine a reasonable way to differentiate the relevant pairs of particles, which are not necessarily in contact, but sufficiently close, from the non-relevant pairs, which are too far from each other. Satake (2004) resolved this problem by using the Dirichlet tessellation (Dirichlet, 1850) of space in a granular assembly. He identified relevant pairs of non-touching particles connected through virtual branches defined by the dual graph of the Dirichlet tessellation, called the Delaunay network (Fig. 2a). Although the concept of a contact was originally defined for touching particles, it can be generalized by introducing a virtual contact, which is defined for relevant pairs of non-touching particles, as a point located on a virtual branch the same distance from the surface of non-touching particles. Hereafter, vc refers to a virtual contact. Additionally, c refers to a generalized contact of both touching and non-touching particles.

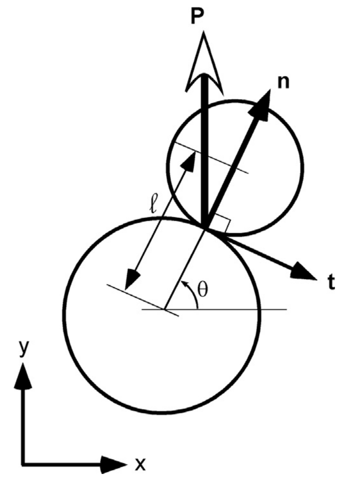

Fig. 1. Contact normal **n**, tangential direction **t**, and contact force **P**.

Hence, Eq. (5) can be formally rewritten as the average over the contacts of both touching and non-touching particles, namely,

$$\boldsymbol{\sigma}' = \frac{1}{V}\sum_{\mathrm{c}} l\mathbf{P} \otimes \mathbf{n} \tag{6}$$

Since non-touching particles do not have contact force, the following condition is always satisfied:

$$\mathbf{P}_{\mathrm{vc}} = \mathbf{0} \tag{7}$$

Following Satake (2004), the macroscopic strain rate is determined by the tensorial average over the relative velocities $\dot{\mathbf{u}}$ between the relevant particle pairs within a representative volume element with volume V as

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{V}\sum_{\mathrm{c}} S\dot{\mathbf{u}} \otimes \mathbf{n} \tag{8}$$

where S denotes the segment areas (or the segment lengths in 2D) of the Dirichlet tessellation (Fig. 2(a)). For simplicity, it is assumed that individual particles do not rotate so that the relative velocities $\dot{\mathbf{u}}$ at the contacts are approximated by the relative velocities of the particle centers.

The macroscopic stress and strain rate in Eqs. (6) and (8) can be written in terms of those due to touching and non-touching particles as

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma}'_{\mathrm{tc}} + \boldsymbol{\sigma}'_{\mathrm{vc}} \tag{9}$$

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}_{\mathrm{tc}} + \dot{\boldsymbol{\varepsilon}}_{\mathrm{vc}} \tag{10}$$