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Quality assessment of soil bearing capacity factor models of shallow foundations

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Abstract

This paper evaluates the uncertainties and quality of bearing capacity factor prediction models of shallow foundations. The development of bearing capacity factor prediction models is a field of extensive research and many different models have been proposed. Sixty models with different modeling approaches such as the analytical model, semi-empirical model, empirical model, finite difference model, upper bound limit model and lower bound with finite element model etc. are connected through a statistical framework that aids in uncertainty quantification and model quality evaluation. First, uncertainty in the estimation of input parameters studies is performed using multivariate information through multiple correlations, in order to determine the parameters that contribute to the uncertainties of the model prediction. Second, the uncertainties of the bearing capacity factor prediction for all models are compared and significant differences are revealed. Due to the consideration of parameter and model uncertainties, a measure for the total variation of the model response is achieved. Results show that the more inaccurate the input parameters are, the more uncertain the quality of the estimated model prediction becomes. With increasing model uncertainty, the quality of the model also decreases. It has been found that the quality of the model decreases as the friction angle increases. A comparison of the models using total model uncertainty appears to be a reliable and economical method for selecting a stochastic model.

Keywords: Bearing capacity; Input parameter; Model quality; Shallow foundation; Uncertainty

1. Introduction

Model and parameter uncertainties are important properties that have to be considered when choosing a reliable and safe model for application. Uncertainty analyses are essential, especially in foundation engineering Keitel et al. (2014) and Motra et al. (2014a). Possible sources of the uncertainties may include inherent variabilities, measurement errors, and modeling (or transformation) uncertainties. More economical geotechnical designs can be achieved by estimating uncertainties through site investigation, particularly by estimating uncertainties in soil shear strength. However, the model remains the main source of uncertainty in geotechnical design.

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Buisman (1940) and Terzaghi (1943) adopted a solution for metal punching proposed by Prandtl (1920, 1921) and applied it to the foundation bearing capacity problem. They defined a three term bearing capacity equation by superposition of the effects of soil cohesion, soil surcharge, and the weight of soil, respectively. For a general case of centric vertical loading of a rigid strip footing (plain strain problem) on a cohesive frictional soil surface with a uniform surcharge of q, the ultimate bearing capacity (q_u) is given as:

$$q_u = cN_c + qN_q + \frac{1}{2} + \gamma BN_\gamma \quad (1)$$

where

c is the soil cohesion;
γ is the unit weight of the soil beneath the foundation;

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B is the footing width;
q is the overburden pressure at the level of the footing base; and
N_c, N_q, and N_γ are the bearing capacity factors for cohesion, overburden, and self-weight of soil, respectively.

For a weightless soil ($\gamma = 0$), Prandtl (1920) and Reissner (1924) developed the following formulae for N_c and N_q and these have been widely accepted:

$$N_c = (N_q - 1)\cot\varphi \tag{2}$$

$$N_q = \tan^2\left(\frac{\varphi}{2} + \frac{\pi}{4}\right)\exp(\pi \tan\varphi) \tag{3}$$

where ϕ is the friction angle.

There is, however, no clear consensus on the best method to define N_γ, and as such, there are many proposed estimation methods. This has become one of the main reasons for disagreement between methods used to estimate q_u, since the value of N_γ for equal values of ϕ can produce large differences, depending on the estimation method used. A closed-form analytical solution for the bearing capacity problem that includes the effects of the unit weight of the soil beneath the footing via the factor N_γ is not possible. Different solutions for N_γ have been developed based on empirical relations, analytical derivations, numerical analyses the finite difference model, the upper bound limit model, or the lower bound with finite element model, etc. Table 1 shows sixty models for estimating N_γ in terms of ϕ, along with the author of each method and the theory on which it is based, as published by Edgar Giovanny (2013).

The selection of the value of the soil friction angle ϕ and the model for N_γ accounts for much of the overall uncertainty in estimating q_u. In addition, the uncertainty of ϕ is estimated by using multivariate information through multiple correlations (Jianye et al., 2012). Furthermore, the practical issue of estimating the mean and standard deviation (first two moments) of the friction angle by incorporating multivariate test index information is addressed for a particular soil. It is possible to quantify the uncertainty of the two moments of the friction angle by incorporating multivariate test index information through a Bayesian analysis (Ching et al., 2012). A general approach is developed to incorporate experimental and sampling uncertainties into probabilistic analyses based on random field methods (Mašín (2015)). Through a comparison with the standard approach which attributes the measured total soil variability to spatial variability, it is shown that consideration of experimental uncertainty may significantly reduce the calculated probability of unsatisfactory performance.

The quality of the prediction, and consequently the reliability of the structural analysis, is mainly dependent on the choice of appropriate models and their coupling. The decision regarding which model to use for a specific application is often based on the experience of the engineer or is made on the basis of experimental data. For simplicity, experimental measurements and model simulations are compared for model uncertainty evaluation. The cost of performing experiments is high and direct measurements of the bearing capacity factor cannot always be obtained experimentally because of limitations in the test procedures. Bayesian methods for model selection are presented for model assessment without measurements using model averaging as a reference (Most, 2011). A method for quantifying the model quality is generally not available without measurement. For this reason, this paper deals with the development of new methodological bases for evaluating the quality of bearing capacity prediction models without measurement.

2. Evaluation of model quality

Evaluation is based on the total uncertainty of the estimated bearing capacity of the soil. The total uncertainty is composed of the parameter and model uncertainty. The uncertainty analysis considers the complexity of the models (epistemic uncertainty) and the influence of uncertain input parameters on the model output (aleatoric uncertainty). A reference model is used in order to evaluate the deterministic differences in the prognosis and therefore, to determine the epistemic uncertainty. Experimental data could be used for this purpose, but there is usually a lack of specific experimental data for the design process of engineering structures. A clearly defined value of q_u cannot always be obtained experimentally, mostly due to the limitations of the test procedures. Therefore, the reference model is fixed in this study as a benchmark. By using a model with the same complexity, it is safe to assume that the accuracy of describing the physical phenomena should also be the same. The model uncertainty of the other reference models can be defined by Eq. (4).

Model uncertainty is defined as the uncertainty associated with the physical and mathematical assumptions and methods that are an intrinsic part of the model formulation and its implementation. Model uncertainty represents the general discrepancy between model prediction and reality. This difference is described by the coefficient of variation of model error $CV_{model,N\gamma}$ or model uncertainty, respectively. The model uncertainty is defined as:

$$CV_{\mathrm{mod}\ el,N_{\gamma i}} = \frac{\left|N_{\gamma_{reference}} - N_{\gamma i}\right|}{1.645} \tag{4}$$

where

i is the considered model and
N_γ is the estimated bearing capacity factor.

In the second step, the stochastic properties of the input parameter have to be defined, the mean and standard deviation (two moments), which leads to the dimensionless coefficient of variation (CV), as well as the type of the distribution function. Common sources of parameter uncertainty include random and systematic measurement uncertainty. Using the Bayesian