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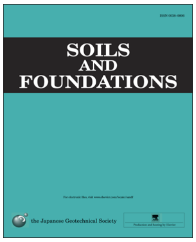

Equation for unimodal and bimodal soil–water characteristic curves

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Abstract

Soil–water characteristic curve (SWCC) data are usually curve fitted with an equation whose parameters were obtained by using optimization procedures. Due to the interdependency of the parameters in the SWCC equation, the parameters are non-unique and thus it is difficult to relate the parameters to other soil properties. This becomes more prominent for bimodal SWCC equations which have more curve-fitting parameters. In order to overcome this limitation, a new equation for both unimodal and bimodal SWCCs is proposed. The new equation uses parameters that can be obtained graphically and no curve-fitting procedures are necessary. Hence, its parameters are unique. The proposed equation was shown to be accurate in representing both unimodal and bimodal SWCCs with several examples.

Keywords: Soil–water characteristic curve; Unimodal; Bimodal; Unsaturated soil

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1. Introduction

It is recognized that soil–water characteristic curves (SWCC) can be unimodal or bimodal (Zhang and Chen, 2005; Li, 2009; Satyanaga et al., 2013; Li et al., 2014) as shown in Fig. 1. A number of equations have been proposed for unimodal and bimodal SWCCs and their parameters have been commonly determined by an optimization method where the SWCC data are curve fitted with a SWCC equation, with the error minimized by iterating the parameters. Leong and Rahardjo (1997) have reviewed a number of unimodal SWCC equations and have found that Van Genuchten (1980) and Fredlund and Xing (1994) equations give the best fit to unimodal SWCC. The Fredlund and Xing (1994) equation in terms of gravimetric water content w is given as follows:

$$w = w_{sat}\frac{C(s)}{\left\{\ln\left[\exp(1)+\left(\frac{s}{a_f}\right)\right]^{n_f}\right\}^{m_f}} \tag{1}$$

$$C(s) = 1 - \frac{\ln\left(1+\frac{s}{\Psi_r}\right)}{\ln\left(1+\frac{10^6}{\Psi_r}\right)} \tag{2}$$

where w_{sat} is the saturated gravimetric water content, s is the matric suction, $C(s)$ is a correction function, Ψ_r is the residual matric suction, a_f, n_f and m_f is the curve fitting parameter. The water content in Eq. (1) can also be in terms of volumetric water content θ_w or degree of saturation S_r. Fredlund et al. (2001) recommended that gravimetric water content be used when plotting soil–water characteristic curve for geotechnical engineering.

The parameters of SWCC equations are usually inter-dependent in controlling the shape of the curve (Gitirana and

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List of symbols

a	Additional curve fitting parameter of Gitirana and Fredlund (2004) equation
a_f, n_f, m_f	Curve fitting parameters in Fredlund and Xing (1994)
a_{fL}, n_{fL}, m_{fL}	Curve fitting parameters related to the macropores in Zhang and Chen (2005)
a_{fs}, n_{fs}, m_{fs}	Curve fitting parameters related to the micropores in Zhang and Chen (2005)
$C(\Psi)$	Function in Fredlund and Xing (1994) equation which ensure gravimetric water content equal to zero when matric suction is 1,000,000 kPa
c_{gi}	Parameter which represent the curvature at the intersection point of Gould et al. (2012) equation
c_{pi}	Parameter which represent the curvature at the intersection point of Pham and Fredlund (2008) equation
c_i	Parameter which represent the curvature at the intersection point of the proposed equation
erf	Error function
$f_1(s)$	Dimensionless SWCC functions which correspond to the macropores
$f_2(s)$	Dimensionless SWCC functions which correspond to the micropores
H	Heaviside function
i	Segment index
j	Data point index
m	Slope of the soil–water characteristic curve
m_i	Slope of segment i
n	Number of segments
n_i, l_i, mL_i, λ_i	curve fitting parameters of the Li (2009) equation
R	Ramp function
R^2	Coefficient of determination
RMSE	Root mean square error
s	Matric suction
s_1	Matric suction when w is equal to w_{sat}
s_{i-}	Matric suction at the point where the curve separates from linear the segment $i-1$
s_i	Matric suction of the intersection point between segment i and segment $i-1$
s_{i+}	Matric suction at the point where the curvature merges into linear segment i
s_m	emerging point of SWCC related to macropores and micropores in bimodal SWCC
S_{si}	Additional curve fitting parameter of Satyanaga et al. (2013) equation
w	Gravimetric water content
w_m	water content of the micropores
w_M	water content of the macropores
w_{sat}	Saturated gravimetric water content
$w_1(s)$	SWCC equation corresponding to the macropores
$w_2(s)$	SWCC equation corresponding to the micropores
z	Number of data points
Ψ_r	Curve fitting parameter related to residual matric suction
Ψ_{rL}	Curve fitting parameter related to residual matric suction of the macropores
Ψ_{rs}	Curve fitting parameter related to residual matric suction of the micropores
χ	Curve fitting parameter in Chin et al. (2010) equation

Fredlund, 2004). Thus, different combinations of parameters may produce the same SWCC. For example, Leong and Rahardjo (1997) showed that the parameter Ψ_r in $C(s)$ is not the residual matric suction as three different values of Ψ_r (3000 kPa, 300 kPa and 30 kPa) may give the same residual matric suction while changing the initial part of SWCC by using the same values of $a_f = 300$ kPa, $n_f = 10$ and $m_f = 0.5$ as shown in Fig. 2. Such interdependency is disadvantageous as correlations of the parameters to other soil properties become problematic and the SWCC equation cannot be predicted using other soil properties.

To overcome this problem, Zhai and Rahardjo (2012) showed that more consistent parameters in Fredlund and Xing (1994) equation can be obtained mathematically while Chin et al. (2010) limit the non-uniqueness of the parameters in Fredlund and Xing (1994) equation for unimodal SWCCs by using a single bounded variable χ in estimating the parameters. Another solution is to directly incorporate more consistent or graphically obtainable parameters into the equation such as the slopes and intersection points between linear segments of the SWCC (Pham and Fredlund, 2008; Gould et al., 2012). Unimodal SWCC equations using these types of parameters are presented in Table 1.

2. Review of bimodal SWCC equations

Some soils exhibit bimodal SWCC due to the presence of both macropores and micropores, and therefore a bimodal pore-size distribution. A bimodal pore-size distribution is commonly observed in soils with bimodal grain size distribution (Durner, 1994; Rahardjo et al., 2004; Satyanaga et al., 2013). Thus, the bimodal SWCC cannot be described using a unimodal SWCC equation. A number of researchers have proposed bimodal SWCC equations. These bimodal SWCC equations can be categorized into three approaches of development. The first approach is the piecewise approach (Smettem and Kirkby, 1990; Wilson et al., 1992; Burger and Shackelford, 2001) where the bimodal SWCC is separated into two unimodal SWCCs corresponding to the macropores and micropores in the soil where the merging point or “junction” is arbitrarily determined (Burger and Shackelford, 2001). Each unimodal curve is then curve-fitted separately. The location of the merging point affects the value of the parameters as different merging points produce different SWCCs and thus the parameters are highly non-unique. The advantage of this

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